UNIT-I Properties of Matter

①

STRESS-STRAIN DIAGRAM

* Consider a wire rigidly fixed at one end and gradually loaded at other end.

* The corresponding strain produced for different loads are noted until the wire breaks down.

* A graph is plotted between strain along X-axis and stress along Y-axis

* It is known as stress-strain diagram.

(i) Hooke's Law:

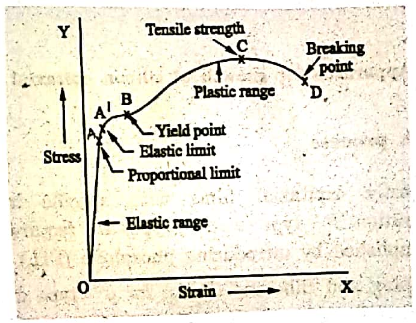
* The portion OA of the curve is a straight line.

* In this region, stress is directly proportional to strain.

* This means that upto OA, the material obey Hooke's law.

* The wire is perfectly elastic.

* The point A is called limit of proportionality or proportionality limit.



(ii) Elastic Limit:

* The stress is further increased till a Point 'A' is reached.

* This point A' lying near A denotes the elastic limit.

* upto this point A', the wire regain its original length if the stress is removed.

* If it is loaded beyond the elastic limit the wire will not restore its original length.

(iii) Yield Point :-

* on further increasing the stress beyond the elastic limit, the curve bends and a point B is reached.

* In this region, A'B a slight increase in Stress produces a larger strain in the material.

* The point B is called yield point.

* The value of the stress at the yield point is called yield strength of the material.

(iv) Permanent Set:

* In the region A'B, if stress is removed the wire will never return to its original length but the wire is said to have taken a permanent set.

(v) Plastic range:

* Beyond B, the strain increases rapidly without any increase in the load. This is called Plastic flow.

vi) Ultimate Strength:

* If the wire is further loaded, a point C is reached after which the wire begins to neck down or flow locally so that its cross Sectional area no longer remains uniform.

* At this point C, the wire begins to thin down at some point and finally breaks.

* At the point C, the value of developed stress is maximum and is called ultimate tensile strength or tensile strength of given material.

vii) Breaking point :-

* The point 'D' is known as breaking point where the wire breaks down completely.

* The stress corresponding to D is called breaking stress.

Material Properties :-

(i) Ductility:

* A material is said to be ductile if it can be readily drawn into wires.

* In terms of stress-strain curve, the materials show ductility behaviour when they are extended beyond yield limit.

* It is the property related to elongation when the material becomes plastic.

* Examples: Gold, Silver, copper etc.

(ii) Malleability :-

* A material is said to be malleable if it can readily be beaten out in the form of thin sheets.

* Malleable material should be soft.

* It should have large elongation for small stress.

* In terms of stress - strain graph, the materials show malleability when they are compres[sed] beyond the yield point.

* Examples : Gold, Silver, Aluminium etc.

(iii) Brittleness :

* Most of the materials first pass through elastic region and then pass through plastic region before they break.

* However there is a type of materials know[n as] a brittle materials which break even before enterin[g] the plastic region.

* A brittle material fractures and breaks int[o] pieces under the influence of large forces but it remains elastic till it breaks.

* Examples : Glass, ceramics and cast iron.

Uses of Stress-Strain diagram:-

* It is used to measure the elastic strength, Yield strength & tensile strength of metals.

* It is used to estimate the working stres[s] and safety factor of an Engineering material.

* The area under the curve in the elastic r[egion] gives the energy required to deform it elasticall[y].

* The area under the curve upto ultimat[e] tensile strength gives the energy required to defo[rm] it plastically.

* It is also used to identify the ductil[e] & brittle material.

Factors affecting elastic Modulus and Tensile Strength :-

The following factors affect the elastic (5) modulus & tensile strength of the materials.

→ Effect of Stress
→ Effect of change in temperature
→ Effect of impurities
→ Effect of hammering, rolling & annealing
→ Effect of crystalline nature.

(i) Effect of Stress :-

The action of large constant stress or repeated number of cycles of stresses acting on a body decreases the elasticity of the body.

(ii) Effect of change in temperature :-

* A change in temperature affects the elastic Properties of a material.

* A rise in temperature usually decreases the elasticity of the material.

* This is due to increase of grain size with rise of temperature, so the distance between atom also increases, the elastic restoring force decreases. Hence decreases the elasticity.

* A carbon filament is highly elastic at normal temperature, it become plastic at high temperature.

* Similarly a decrease in temperature will increase the elastic property.

* Lead is not very good elastic material. But at low temperature, it becomes very good elastic material.

* In some cases like Invar steel, the elasticity is not affected by change in temperature.

(iii) Effect of Impurities :-

* The elastic property of a material is either increased or decreased due to the addition o impurities.

* It depends on the elastic or plastic properties of impurities added.

* If the impurity has more elastic than th material, it increases the elasticity.

* If the impurity has less elastic than the material, it decreases the elasticity.

* If minute quantities of carbon is added wi molten iron, the elastic properties of iron are incre If more carbon is added, its elastic properties decre

* The addition of potassium or copper in gold increases the elastic property of gold.

(iv) Effect of Hammering, Rolling & Annealing :-

* while hammered or rolled, crystal grains brea into smaller grains resulting in increase of elastic properties.

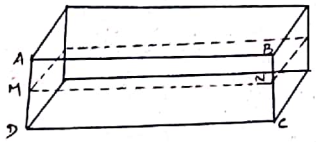
* while annealing, constituent crystals are uniform oriented and form larger crystal grains. This result decrease in elastic properties.

(v) Effect of crystalline Nature :-

* For a given metal, the modulus of elasticity is more when it is in single crystal form & in polycrystalline state, the modulus of elasticity is small.

BEAM :-

A beam is a rod or bar of uniform cross section whose length is very much greater than its thickness.



ASSUMPTIONS :-

While studying about the bending of beams, the following assumptions has to be made.

* The length of the beam should be large compared to other dimensions.

* The forces applied should be large compared to weight of the beam.

* The cross section of the beam remains constant.

* The shearing stresses are negligible.

* The curvature of the beam is very small.

Bending of beam :-

* Consider a beam which is bent into an arc of a circle by the application of a load.

* This beam is made up of a large number of thin plane layers one above the other.

* Taking a longitudinal section ABCD of the bent beam, the layers in the upper half are elongated while those in the lower half are compressed.

* In the middle, there is a layer (MN) which is not elongated or compressed due to bending of beam.

* This layer is called neutral surface and the line MN at which the neutral layer intersects the plane of bending is called neutral axis.

* Therefore the length of the layer increases or decreases in proportion to the distance from the neutral axis.

* The layers below MN are compressed and those above MN are elongated.

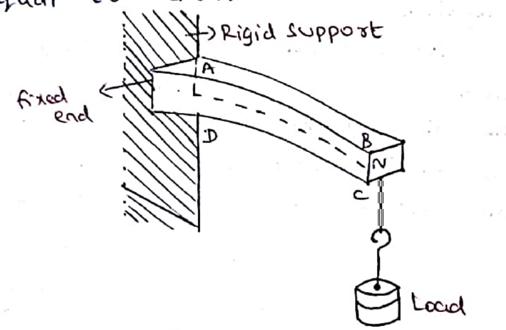
* There are pair of layers one above MN and one below MN experiencing same forces of elongation and compression due to bending.

* Each pair of layers forms a couple.

* This couple is called internal couple.

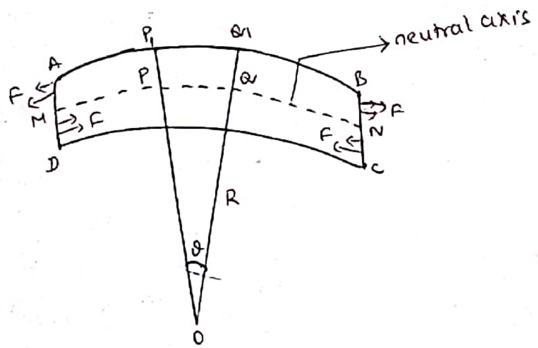
* The resultant of the moments of all these internal couples are called Internal Bending Moment.

* In equilibrium condition, Internal bending moment is equal to external bending moment.



BENDING MOMENT OF A BEAM :-

Consider a portion ABCD of a bent beam as shown in figure.



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* P and Q are two points on the neutral axis MN.

* R is the radius of curvature of the neutral axis and θ is the angle subtended by bent beam at its centre of curvature O. ($\angle POQ = \theta$)

* Consider two corresponding points P_1 and Q_1 on a parallel layer at a distance 'x' from the neutral axis.

From the figure,

$$PQ = R \times \theta \quad \text{—— (1)}$$

Corresponding length on the parallel layer

$$P_1Q_1 = (R+x)\,\theta \quad \text{—— (2)}$$

Increase in length of $P_1Q_1 = P_1Q_1 - PQ$

$$= (R+x)\theta - R\theta$$

$$= R\theta + x\theta - R\theta$$

$$= x\theta \quad \text{—— (3)}$$

Before bending $P_1Q_1 = PQ$

∴ Longitudinal strain produced $= \dfrac{\text{Increase in length}}{\text{Original length}}$

$$= \frac{x\theta}{R\theta} = \frac{x}{R} \quad \text{—— (4)}$$

If Y is the Young's modulus of the material,

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

∴ Longitudinal stress = Y × Longitudinal strain

$$= Y \times \frac{x}{R} \quad \text{—— (5)}$$

If δA is the area of cross section of the filament, then

Force acting on area δA = Stress × area

$$= \frac{Yx}{R} \times \delta A$$

$\therefore$ Moment of force = force × Perpendicular distance

$$= \frac{Yx}{R} \times \delta A \times x$$

$$= \frac{Y}{R} \delta A x^2$$

$\therefore$ The sum of moments of forces acting on all the filaments is given by

$$= \Sigma \frac{Y}{R} \delta A x^2$$

$$= \frac{Y}{R} \Sigma \delta A x^2$$

Internal bending Moment $= \frac{YI}{R}$ $[\because I = \Sigma \delta A x^2]$

{I is called geometrical moment of inertia of the cross section of the beam}

NOTE:

(i) For a rectangular beam of breadth b and thickness d is

$$I = \frac{bd^3}{12}$$

$\therefore$ Internal bending moment $= \frac{Ybd^3}{12R}$

(ii) For a beam of circular cross section

$$I = \frac{\pi r^4}{4}$$

$\therefore$ Internal bending moment $= \frac{Y\pi r^4}{4R}$

where r is the radius of the rod.

CANTILEVER:-

It is a beam fixed horizontally at one end and loaded at the other end.

Expression for depression Produced in the cantilever:-

* Consider a cantilever of length l fixed at end A and loaded at the free end B by a weight W.
* The end B is depressed to B'.
* AB is the neutral axis.
* BB' represents the vertical depression at free end.
* Consider the section P at a distance x from the fixed end A.
* It is at a distance (l−x) from the loaded end B'.
* Consider the equilibrium of portion PB', there is a force of reaction W at P.

$\therefore$ External bending moment = W × PB' = W(l−x) — ①

Internal bending moment $= \frac{YI}{R}$ — ②

where Y → Young's modulus of cantilever

I → Moment of Inertia

R → Radius of curvature of neutral axis.

At equilibrium position,

External bending moment = Internal bending moment

$$W(l-x) = \frac{YI}{R} \quad \text{— ③}$$

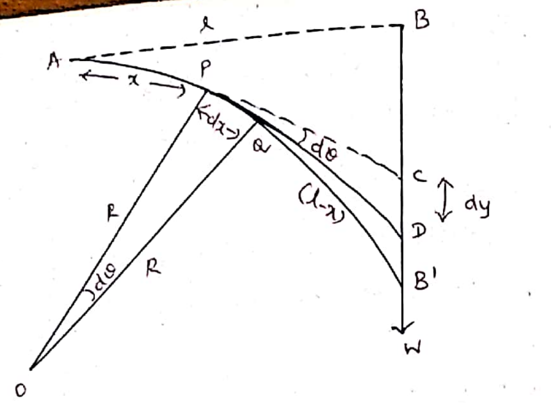
Q is another point at a distance dx from P

$$\therefore PQ = dx \quad \text{— ④}$$

O is the centre of curvature of arc PQ

PO = R and ∠POQ = dθ

$$\therefore dx = R\,d\theta \quad \text{— ⑤}$$



The tangents are drawn at P & Q meeting the BB' at C & D.

$\therefore$ vertical depression $CD = dy = (l-x)d\theta$ — ⑥

From eqn ⑤ & ⑥ we can write

$$\frac{dx}{dy} = \frac{R\,d\theta}{(l-x)d\theta} = \frac{R}{l-x}$$

$$\therefore R = \frac{(l-x)\,dx}{dy} \quad — ⑦$$

Sub eqn ⑦ in eqn ③

$$W(l-x) = \frac{YI \cdot dy}{(l-x)dx}$$

Rearranging

$$dy = \frac{W(l-x)^2 dx}{YI}$$

$\therefore$ Total depression at the free end is given by

$$y = \int_0^l dy$$

$$= \int_0^l \frac{W}{YI}(l-x)^2 dx$$

$$= \frac{W}{YI}\int_0^l (l^2+x^2-2lx)\,dx$$

$$= \frac{W}{YI}\left[l^2x + \frac{x^3}{3} - \frac{2lx^2}{2}\right]_0^l$$

$$= \frac{W}{YI}\left[l^3 + \frac{l^3}{3} - l^3\right]$$

$$= \frac{W}{YI} \times \frac{l^3}{3}$$

$$y = \frac{Wl^3}{3YI}$$

The Youngs modulus of the cantilever is determined using the value of depression produced in the cantilever.

$$\therefore Y = \frac{Wl^3}{3Iy}$$

For a beam of rectangular cross section,

$I = \frac{bd^3}{12}$ where b is breadth & d is thickness of beam.

The weight $W = mg$ where M is the mass suspended at the free end and g is acceleration due to gravity.

$$\therefore Y = \frac{mgl^3}{3\left(\frac{bd^3}{12}\right)y}$$

$$\boxed{\therefore Y = \frac{4Mgl^3}{bd^3y} \; N/m^2}$$

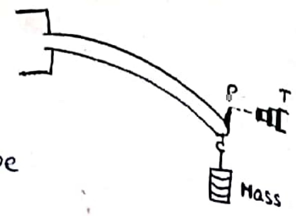
Experimental determination of Young's modulus Using Cantilever:-

* The given bar is fixed rigidly at one end & a weight hanger is suspended at other end.
* A pin is fixed vertically using wax at the free end of the beam.
* A travelling microscope (T) is focussed on the pin.

* The microscope is ... with the tip of the horizontal cross wire coincides with the tip of the ...

* The initial reading in the microscope on the vertical scale is noted.

* A suitable mass M is placed on the hanger.



* The reading in the microscope is again noted.

* The difference between two readings gives the depression y corresponding to load M.

* The experiment is repeated by increasing the values of M in steps of 50 gms.

* Then the experiment is also repeated by decreasing the weights.

* The observations are tabulated.

* from these observations, mean y is obtained.

* The length of the beam, breadth b (using vernier calipers) & thickness d (using screw guage) are measured.

* By using the above values, youngs modulus of the beam is determined using

$$Y = \frac{4Mgl^3}{bd^3y} \ N/m^2$$

Load (gm)	Microscope Reading			Mean depression for M kg (cm)
	Increasing Load (cm)	Decreasing Load (cm)	Mean (cm)	
W				
W+50				
W+100				
W+150				
W+200				
W+250				

Mean y =

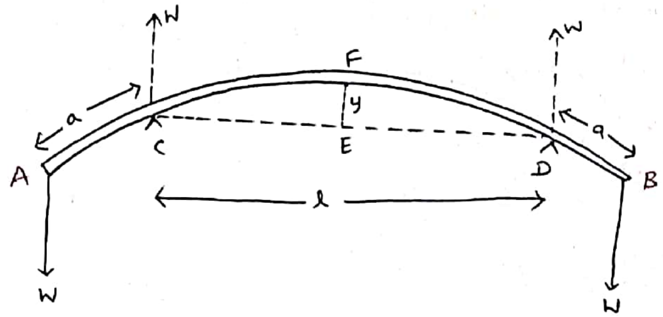
Uniform bending (Theory and Experiment) :-

Definition:

If the beam is loaded uniformly on its both ends, bending of the beam forms an arc of a circle. The elevation is produced in the beam. This type of bending is called uniform bending.

Theory:

* Consider a beam AB arranged horizontally on two knife edges C & D symmetrically so that AC = BD = a.



* The beam is loaded with equal weights W at each ends A & B.

* The reactions on the knife edges at C & D are equal to W and they are acting vertically upward.

∴ External Bending moment on the part AF of the beam about F is

$= W \times AF - W \times CF$

$= W(AF - CF)$

$= W(AC)$

$= Wa$ —— ①

We know that Internal bending moment $= \frac{YI}{R}$ — ②

Where Y → Young's modulus

I → Moment of Inertia

R → Radius of curvature

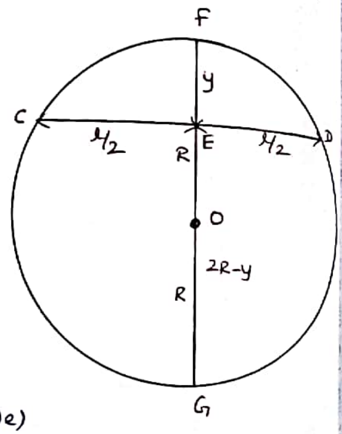
In equilibrium position,

External bending moment = bending moment

$$Wa = \frac{YI}{R} \quad \text{--- (3)}$$

Since for a given value of W, the values of a, Y, and I are constant. R is constant so that the beam bends uniformly into an arc of a circle of radius R.

CD = l & y is the elevation of the midpoint E of the beam so that y = EF.



From the property of circle,

$$EF \times EG = CE \times ED$$

$$y(2R-y) = \frac{l}{2} \times \frac{l}{2}$$

$$2Ry - y^2 = \frac{l^2}{4}$$

$$2Ry = \frac{l^2}{4} \quad (\because y^2 \text{ is negligible})$$

$$R = \frac{l^2}{8y}$$

$$\frac{1}{R} = \frac{8y}{l^2} \quad \text{--- (4)}$$

Sub eqn (4) in eqn (3)

$$Wa = YI \times \frac{8y}{l^2}$$

$$y = \frac{Wal^2}{8IY}$$

$$Y = \frac{Wal^2}{8Iy}$$

If the beam is rectangular cross section,

$I = \frac{bd^3}{12}$, b - breadth & d - thickness of beam.

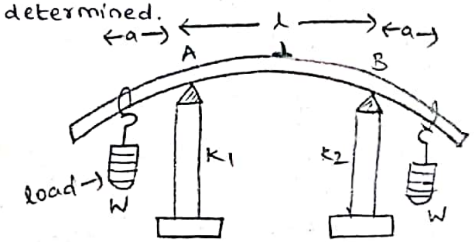
If M is the mass, the weight W = Mg.

$$\therefore Y = \frac{Mgal^2}{8\left(\frac{bd^3}{12}\right)y}$$

$$Y = \frac{3Mgal^2}{2bd^3y} \ N/m^2$$

Experiment :-

* A rectangular bar AB of uniform cross section is supported horizontally on two knife edges A & B.
* Two weight hangers of equal masses are suspended from the ends of the beam.
* The pin is fixed vertically at the mid point of the beam.
* A microscope is focussed on the tip of the pin.
* Initial reading in the microslope in the vertical scale is noted.
* Equal weights are added to both hanger simultaneously & the reading in the microslope is noted.
* The experiment is repeated for decreasing order of equal masses.
* The observations are tabulated & mean elevation y at mid point of the bar is determined.



* The length of the bar between the knife edges 'l' is measured.
* The distance of one of the weight hanger from the nearest knife edge 'a' is noted.
* The breadth 'b' & thickness 'd' is noted. Using Vernier caliper & screw gauge.

Load (gm)	Microscope Reading			Mean elevation (y) for M kg
	Increase Load (cm)	Decrease Load (cm)	Mean (cm)	
W				
W+50				
W+100				
W+150				
W+200				
W+250				
			Mean(y)	

Young's Modulus is

$$Y = \frac{3}{2}\frac{Mgal^2}{bd^3y} \ N/m^2$$

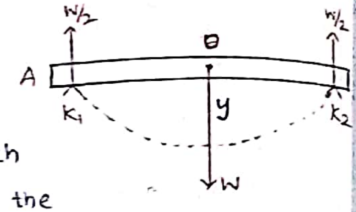
Non-uniform bending - Theory and experiment :-

Definition:-

If the beam is loaded at its mid point, the depression produced does not form an arc of a circle. This type of bending is called non-uniform bending.

Theory:-

* Consider a uniform cross sectional beam AB of length l arranged horizontally on two knife edges K_1 & K_2 near the end A & B.



* A weight W is applied at the midpoint 'O' of the beam.
* The reaction force at each knife edge is equal to $\frac{W}{2}$ in the upward direction.
* y is the depression at the midpoint O.
* The bent beam is considered to be equivalent to two inverted cantilevers fixed at O each of length $\frac{l}{2}$ and loaded at K_1 & K_2 with weights $\frac{W}{2}$.
* In the case of cantilever of length l & load W, the depression $= \frac{Wl^3}{3IY}$
* Hence for cantilever of length $\frac{l}{2}$ & load $\frac{W}{2}$ the depression is

$$y = \frac{\left(\frac{W}{2}\right)\left(\frac{l}{2}\right)^3}{3IY} = \frac{\frac{W}{2} \times \frac{l^3}{8}}{3IY}$$

$$y = \frac{Wl^3}{48IY} \quad ; \quad Y = \frac{Wl^3}{48Iy}$$

If the beam is rectangular cross section,

$I = \frac{bd^3}{12}$, b-breadth & d-thickness of beam.

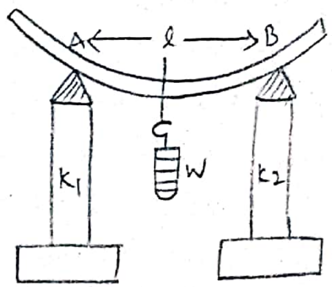
If M is the mass, weight $W = Mg$.

$$\therefore Y = \frac{Mgl^3}{48\left(\frac{bd^3}{12}\right)y}$$

$$\boxed{Y = \frac{Mgl^3}{4bd^3y}} \; N/m^2$$

Experiment:-

* The given beam AB of rectangular cross section is arranged horizontally on two knife edges K_1 & K_2 near A & B.
* A weight hanger is suspended & a pin is fixed vertically at mid point O.
* A microscope is focussed on tip of the pin.
* The initial reading on the vertical scale of the microscope is taken.
* A suitable mass M is added to the hanger, hence the beam is depressed.
* The cross wire is adjusted to coincide with the tip of the pin & the reading of the microscope is noted.
* The depression corresponds to mass M is found.



* The experiment is repeated by increasing & decreasing the mass step by step & readings are noted.
* The average value of depression is found.
* The length of the bar 'l' between knife edges is measured.
* The breadth 'b' and thickness 'd' is noted using vernier caliper & screw gauge.

Load (gm)	Microscope Reading			Mean depression (y) for M kg
	Increase Load (cm)	Decrease Load (cm)	Mean (cm)	
W				
W+50				
W+100				
W+150				
W+200				
W+250				
			Mean (y)	

Young's Modulus is

$$Y = \frac{Mgl^3}{4bd^3y} \; N/m^2$$

I-Shape Girder:-

Definition:-

The girders with upper and lower section broadened and the middle section tapered so tha it can withstand heavy loads over it is called I shape girders. Since it looks like letter I, it is named as I shape girder.

Explanation:-

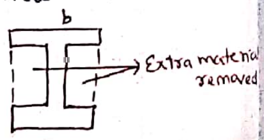
* The depression y at the centre of a be of length l, breadth b and thickness d under a lo Mg at its midpoint is

$$y = \frac{Mgl^3}{4bd^3Y}$$

* Hence to reduce bending for a given load, Y sh be large, b & d of beam must be large and l sho be as small as possible.
* Since the depression is inversely proportional to the depression can be reduced more effectively by increasing the thickness rather than increasing the bre
* But on increasing the thickness, unless the load is at the centre, the beam may bend as sh in figure. This is called buckling of beam.

Bu

* To prevent buckling, a large load bearing surface is required.
* Hence the beam is designed to have large thickness to minimise bending and a large load bearing surface to prevent buckling.
* Therefore the cross sectional view of the beam as shown in figure is preferred.



Advantages:-

* Low cost
* It provides high bending moment
* It is made up of steel having high Young's modulus.
* More stability & more stronger.
* High durability.

Applications:-

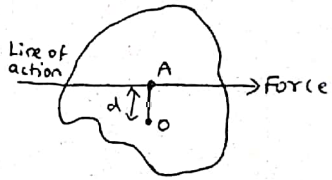
* It is used in construction of bridges over rivers.
* It is used in I-section railway tracks.
* It is used in construction of iron beams to support the bridges for heavy vehicles.
* It is used in construction of dams.
* It is used as supporting beams for ceilings in the construction of buildings.

Moment, Couple and Torque :-

(i) Moment of a force :-

The moment of a force about a point is defined as the product of the magnitude of force and the perpendicular distance from the point to the line of action of force.

Let F be the force acting on a body at A.



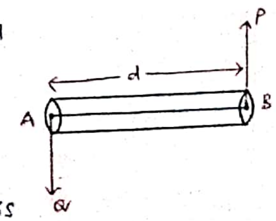
Then the moment of force F about 'o' is $M = F \times d$.

Where $d \rightarrow$ perpendicular distance from the point 'o' to the line of action of force F.

(ii) Couple :-

A couple constitutes a pair of two equal and opposite forces acting on a body in such a way that the lines of action of the two forces are not in the same straight line.

Let P and Q be the two equal and opposite forces acting on the body AB.



These two forces form a couple and the moment of the couple about A is M_A and about B is M_B, then

$$\text{Couple} = M_A = M_B = P \times d = Q \times d.$$

(iii) Torque (τ) :-

Torque of a force with respect to a fixed point is defined as the product of force and the perpendicular distance of the fixed point from the line of action of force. It has the tendency to rotate the body about the axis passing through the fixed point.

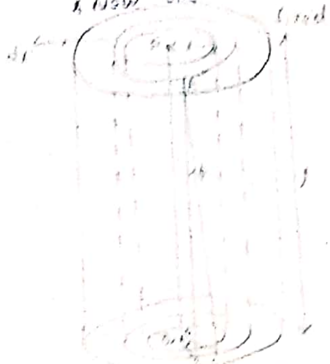
$$\text{Torque}(\tau) = F \times d.$$

Application of elasticity to torsion of wires or cylinders or shafts:-

The concepts of elasticity can be applied to the torsion of wires or cylinders & torsion pendulum.

Twisting couple on a wire:

* Consider a cylindrical wire of length l, radius r fixed at one end.
* It is twisted through an angle θ by applying couple to its lower end.
* Now the wire is said to be under torsion.



* Due to elastic property of the wire, an internal restoring couple is set up inside the wire.
* It is equal and opposite to the external twisting couple.
* The cylinder is imagined to consist of a large number of thin hollow coaxial cylinders.
* Consider one such cylinder of radius x & thickness dx.
* AB is a line parallel to PQ on the surface of this cylinder.
* As the cylinder is twisted, the line AB is shifted to AC through an angle $BAC = \phi$.

Shearing strain or Angle of shear $= \phi$

Angle of twist at free end $= \theta$

From the figure,

$$BC = x\theta = l\phi$$

$$\phi = \frac{x\theta}{l}$$

$$\text{Rigidity modulus } n = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

Shearing stress $= n \times$ shearing strain $= n\phi$

$$= \frac{nx\theta}{l} \quad \text{———} \; (1)$$

$$\text{But shearing stress} = \frac{\text{Shearing force}}{\text{Area over which the force acts}}$$

Shearing force $=$ Shearing stress $\times$ area over which the force acts.

$$\left.\begin{matrix}\text{Area over which the} \\ \text{force acts}\end{matrix}\right\} = \pi(x+dx)^2 - \pi x^2$$

$$= \pi(x^2 + 2x\,dx + dx^2) - \pi x^2$$

$$= \pi x^2 + \pi 2x\,dx + \pi dx^2 - \pi x^2$$

$$= 2\pi x\,dx \quad (\because dx^2 \text{ - small, so neglected})$$

$$\text{Hence shearing force } F = \frac{nx\theta}{l} 2\pi x\,dx$$

$$= \frac{2\pi n\theta}{l} x^2\,dx \quad \text{———} \; (2)$$

Moment of this force about the axis PQ of the cylinder is $=$ force $\times$ perpendicular distance

$$= \frac{2\pi n\theta}{l} x^2\,dx \times x$$

$$= \frac{2\pi n\theta}{l} x^3\,dx \quad \text{———} \; (3)$$

The moment of the force acting on the entire cylinder of radius is obtained by integrating between $x=0$ & $x=r$.

$$\therefore \text{Twisting couple } C = \int_0^r \frac{2\pi n\theta}{l} x^3\,dx.$$

$$= \frac{2\pi n\theta}{l} \int_0^r x^3 dx.$$

$$= \frac{2\pi n\theta}{l} \left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{2\pi n\theta}{l} \left(\frac{r^4}{4_2} - 0\right)$$

$$\boxed{C = \frac{\pi n\theta r^4}{2l}} \quad \text{—— (4)}$$

If $\theta = 1$ radian,

Twisting couple per unit twist is

$$\boxed{C = \frac{\pi n r^4}{2l}} \quad \text{—— (5)}$$

This twisting couple required to produce a twist of unit radian in the cylinder is called torsional rigidity for material of the cylinder.

Hollow cylinder:-

For a hollow cylinder of same length l & inner radius r_1 and outer radius r_2.

$$\left.\begin{array}{l}\text{Twisting couple of the} \\ \text{cylinder } C\end{array}\right\} = \int_{r_1}^{r_2} \left(\frac{2\pi n\theta}{l}\right) x^3 dx.$$

From (3)

$$= \frac{2\pi n\theta}{l} \left[\frac{x^4}{4_2}\right]_{r_1}^{r_2}$$

$$= \frac{\pi n\theta}{2l} \left[r_2^4 - r_1^4\right]$$

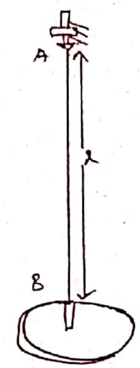
Twisting couple per unit twist of the cylinder is

$$\boxed{c' = \frac{\pi n}{2l} (r_2^4 - r_1^4)}$$

TORSION PENDULUM - THEORY AND EXPERIMENT:- (25)

* A circular metallic disc suspended using a thin wire that executes torsional oscillation is called torsional pendulum.

* A Simple pendulum executes linear oscillation.



Description:-

* A torsional pendulum consist of a metal wire suspended vertically with the upper end fixed.

* The lower end of the wire is connected to the centre of a heavy circular disc.

* When the disc is rotated by applying a twist, the wire is twisted through an angle θ.

Then the restoring couple setup in the wire is $= C\theta$ —— (1)

where $C \rightarrow$ couple per unit twist.

If the disc is released, it oscillates with angular velocity $\frac{d\theta}{dt}$ in the horizontal plane about the axis of the wire.

These oscillations are called torsional oscillations.

If $\frac{d^2\theta}{dt^2}$ is the angular acceleration produced in the disc and I is the moment of inertia of the disc, then

$$\text{Applied couple} = I \frac{d^2\theta}{dt^2} \quad \text{—— (2)}$$

In Equilibrium condition,

Applied couple = Restoring couple.

$$I \frac{d^2\theta}{dt^2} = C\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{C}{I}\theta \quad \text{—— (3)}$$

This equation represents simple harmonic motion which shows that angular acceleration $\left(\frac{d^2\theta}{dt^2}\right)$ is proportional to angular displacement θ and is always directed towards the mean pos[ition] being

Hence the motion of the disc being simple harmonic motion, the time period of the oscillation is given by

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$= 2\pi\sqrt{\frac{\theta}{C/I \times \theta}}$$

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Uses of Torsional Pendulum :-

* To determine rigidity modulus of the wire
* To determine moment of Inertia of disc
* To determine moment of inertia of irregular
* To determine

Determination of Rigidity Modulus of the Wire :-

The rigidity modulus of the wire is determined by using

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Experiment :-

* A circular disc is suspended by a thin wire, rigidity modulus is to be determined.
* The top end of the wire is fixed in a vert[ical] support.
* The disc is then rotated about its cent[re] through a small angle & set it free.
* It executes torsional oscillations.
* The time taken for 20 complete oscillations is noted. (27)
* The experiment is repeated and the mean time period (T) of oscillation is noted.
* The length of the wire (l) is noted.
* This length is then changed by about 10cm and then the experiment is repeated.
* The readings for five or six different lengths of wire are measured.
* The disc is removed & its mass & diameter are measured.

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Squaring

$$T^2 = \frac{4\pi^2 \times I}{C}$$

Sub $C = \frac{\pi n r^4}{2l}$ we get

$$T^2 = \frac{4\pi^2 I \times 2l}{\pi n r^4}$$

Rearranging,

$$n = \frac{8\pi I}{r^4}\left(\frac{l}{T^2}\right)$$

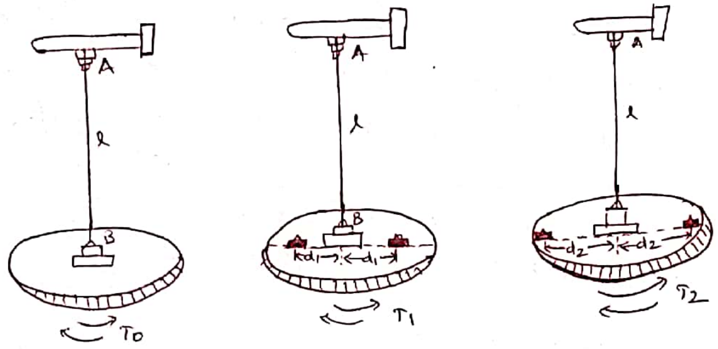
where $I \rightarrow$ moment of Inertia $= \frac{MR^2}{2}$

$M \rightarrow$ Mass of the disc

$R \rightarrow$ Radius of the disc.

Rigidity Modulus by Torsion Pendulum (Dynamic Torsion Method) :-

The torsion pendulum consist of a steel or brass wire with one end fixed in an adjustable chuck & the other end to the centre of a circular disc.



* The experiment consist of 3 parts.

* First the disc is set into torsion oscillation without any cylindrical masses on the disc.

* The mean period of oscillation 'T_0' is found

$$T_0 = 2\pi\sqrt{\frac{I_0}{c}}$$

where I_0 - moment of inertia of disc

Squaring, $$T_0^2 = \frac{4\pi^2 I_0}{c} \quad —①$$

* Two equal cylindrical masses (200gm) are placed symmetrically along a diameter of disc at equal distance d_1 on the two sides of the centre of disc. (nearer).

* Mean time period T_1 is found.

$$T_1 = 2\pi\sqrt{\frac{I_1}{c}}$$

Squaring, $$T_1^2 = \frac{4\pi^2 I_1}{c} \quad —②$$

where I_1 - moment of inertia of whole system (disc + mass)

c - couple per unit twist

Then by parallel axis theorem, the moment of inertia of whole system is

$$I_1 = I_0 + 2i + 2md_1^2 \quad —③$$

sub eqn ③ in eqn ②

$$\therefore T_1^2 = \frac{4\pi^2}{c}(I_0 + 2i + 2md_1^2) \quad —④$$

where $i \rightarrow$ moment of inertia of each mass about an axis passing through the centre.

Now, two cylindrical masses are placed symmetrically at equal distance d_2 from the axis of the wire (farthest).

Mean time period T_2 is

$$T_2 = 2\pi\sqrt{\frac{I_2}{c}}$$

Squaring $$T_2^2 = \frac{4\pi^2 I_2}{c}$$

||ly $$T_2^2 = \frac{4\pi^2}{c}[I_0 + 2i + 2md_2^2] \quad —⑤$$

Now $$I_2 - I_1 = I_0 + 2i + 2md_2^2 - I_0 - 2i - 2md_1^2$$
$$= 2m(d_2^2 - d_1^2)$$

Sub ④ from ⑤

$$T_2^2 - T_1^2 = \frac{4\pi^2}{c}[I_0 + 2i + 2md_2^2] - \frac{4\pi^2}{c}[I_0 + 2i + 2md_1^2]$$
$$= \frac{4\pi^2}{c}(I_2 - I_1) \quad —⑥$$

$$\frac{①}{⑥} \Rightarrow \frac{T_0^2}{T_2^2 - T_1^2} = \frac{4\pi^2 I_0}{c} \times \frac{c}{4\pi^2(I_2 - I_1)}$$
$$= \frac{I_0}{I_2 - I_1}$$

Sub the value of $I_2 - I_1$ in eqn ⑦

$$\frac{T_0^2}{T_2^2 - T_1^2} = \frac{I_0}{2m(d_2^2 - d_1^2)}$$

$$\boxed{I_0 = \frac{2m(d_2^2 - d_1^2)T_0^2}{T_2^2 - T_1^2}}$$

Thus the moment of inertia of the disc is calculated.

Calculation of Rigidity Modulus of the Wire:-

We know that

$$C = \frac{\pi n r^4}{2l}$$

Sub the value of C in eqn ⑥

$$T_2^2 - T_1^2 = \frac{4\pi^2}{\frac{\pi n r^4}{2l}} 2m(d_2^2 - d_1^2)$$

$$= \frac{4\pi^2 \times 2l \times 2m(d_2^2 - d_1^2)}{\pi n r^4}$$

$$\boxed{n = \frac{16\pi l m (d_2^2 - d_1^2)}{(T_2^2 - T_1^2) r^4} \quad N/m^2}$$

Thus the rigidity modulus of the wire is determined.

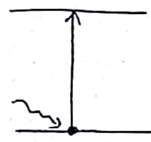
LASER

Einstein's A and B Coefficients :-

When light radiation is incident on the atoms, three different processes takes place.

(a) Stimulated absorption :-

The atoms in the lower energy state E_1 absorbs radiation and is excited to higher energy level E_2. This is called stimulated absorption.



The rate of stimulated absorption is directly proportional to the number of atoms in lower energy state and energy density.

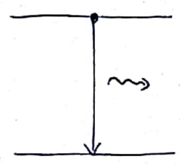
$$N_{ab} \propto N_1 Q$$

$$N_{ab} = B_{12} N_1 Q \quad \text{—— ①}$$

This is upward transition.

(b) Spontaneous Emission :-

The atoms in the excited state E_2 return to lower energy state E_1 by emitting a photon of energy $h\nu$ without any energy. This is called spontaneous emission.



The rate of spontaneous emission is directly proportional to the number of atoms in excited state.

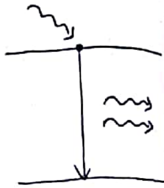
$$N_{sp} \propto N_2$$

$$N_{sp} = A_{21} N_2 \quad \text{—— ②}$$

This is downward transition.

(c) Stimulated Emission :-

The atoms in the excited state absorbs radiation and is return to ground stat by emitting two photons. This is called stimulat emission.



The rate of stimulated emission is directly proportional to number of atoms in excited state and energy density.

$$N_{st} \propto N_2 Q$$

$$N_{st} = B_{21} N_2 Q \quad \text{——— (3)}$$

This is downward transition.

The proportionality constants A_{21}, B_{12} & B_{21} called Einstein coefficients.

Under equilibrium condition, the number of upward and downward transition are equal.

$$B_{12} N_1 Q = A_{21} N_2 + B_{21} N_2 Q \quad \text{——— (4)}$$

$$B_{12} N_1 Q - B_{21} N_2 Q = A_{21} N_2$$

$$Q [B_{12} N_1 - B_{21} N_2] = A_{21} N_2$$

$$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \quad \text{——— (5)}$$

$\div$ by $B_{21} N_2$

$$Q = \frac{\frac{A_{21}}{B_{21}} \cdot \frac{N_2}{N_2}}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - \frac{B_{21} N_2}{B_{21} N_2}}$$

$$Q = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12}}{B_{21}} \cdot \frac{N_1}{N_2} - 1} \quad \text{——— (6)}$$

From Boltzmann distribution equation, $\frac{N_1}{N_2} = e^{h\nu/kT}$

$$\therefore Q = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/kT} - 1} \quad \text{——— (7)}$$

Planck's radiation formula is given by

$$Q = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1} \quad \text{——— (8)}$$

comparing eqn (7) & (8)

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}} \; ; \; \frac{B_{12}}{B_{21}} = 1 \; ; \; \boxed{B_{12} = B_{21}}$$

$$A_{21} = A \;\&\; B_{12} = B_{21} = B$$

These A & B are called Einstein coefficients.

Ratio of magnitudes of Stimulated & spontaneous Emission :-

From eqn (2) + (3)

$$\frac{(3)}{(2)} \Rightarrow \frac{N_{st}}{N_{sp}} = \frac{B_{21} N_2 Q}{A_{21} N_2}$$

$$\frac{N_{st}}{N_{sp}} = \frac{B_{21}}{A_{21}} Q \quad \text{——— (9)}$$

Sub eqn (7) in (9)

$$\frac{N_{st}}{N_{sp}} = \frac{B_{21}}{A_{21}} \cdot \frac{A_{21}}{B_{21}} \cdot \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/kT} - 1}$$

Since $B_{12} = B_{21}$

$$\frac{N_{st}}{N_{sp}} = \frac{1}{e^{h\nu/kT} - 1} \quad \text{——— (10)}$$

Comparing (9) & (10)

$$\frac{N_{st}}{N_{sp}} = \frac{1}{e^{h\nu/kT} - 1} = \frac{B_{21}}{A_{21}} Q$$

Simply we can write

$$\boxed{R = \frac{B_{21}}{A_{21}} Q}$$

Conclusion :-

* Stimulated emission is greater than spontaneous emission.

UNIT 2 – WAVES AND OPTICS

LASER

Nd-YAG LASER:

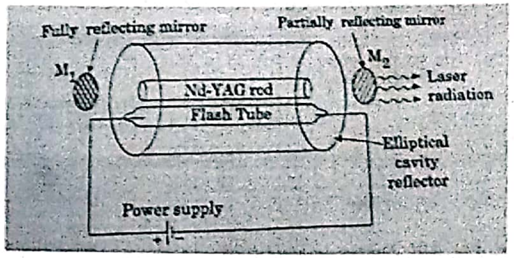
- Nd-YAG laser is a Neodymium based laser.
- Nd stands for Neodymium and YAG stands for Yttrium Aluminium Garnet ($Y_3 Al_5O_{12}$).
- It is a four level solid state laser.

Principle:

- The active medium Nd-YAG rod is optically pumped by Krypton flash tube.
- The neodymium ions (Nd^{3+}) are raised to excited levels.
- During transition from metastable state to ground state, laser beam of wave length 1.064 µm is emitted.

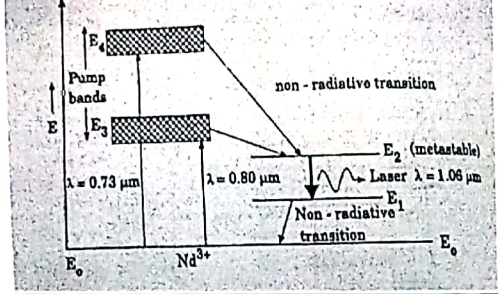
Construction:

- A small amount of Yttrium ions are replaced by Neodymium ions in the active medium.
- The Nd-YAG crystal is cut into a cylindrical rod.
- The Nd-YAG rod and a krypton flash tube are kept in an elliptical cavity.
- The optical resonator is formed by 100% reflecting mirror M_1 on one side and partial reflecting mirror M_2 on other side.



Working:

- The krypton flash tube is switched on and the light is fall on the laser rod.
- The neodymium ions are excited from the ground state E_0 to upper levels E_3 & E_4 by absorbing wavelength of 0.7µm and 0.8µm.
- These ions will go to E_2 by Non-radiative transition.
- Hence population inversion is achieved in metastable state E_2.
- So Neodymium ions make a transition from E2 to E1 by emitting a photon.
- The photons travel back and forth between two mirrors and the photos will multiply.
- After enough strength is reached, laser light of wavelength 1.064µm is emitted through partial reflecting mirror.



Advantages:

- High energy output.
- Easy to achieve population inversion.

Disadvantages:

The energy levels of neodymium ions are complicated.

Applications:

- It is used in welding, drilling etc.
- It is used in endoscopy, urology and neurosurgery

SEMICONDUCTOR DIODE LASER (INJECTION LASER):

There are two types of Semiconductor diode laser.

- ✓ Homo junction Semiconductor diode laser
- ✓ Hetero junction Semiconductor diode laser

Homo junction Semiconductor diode laser:

Definition:

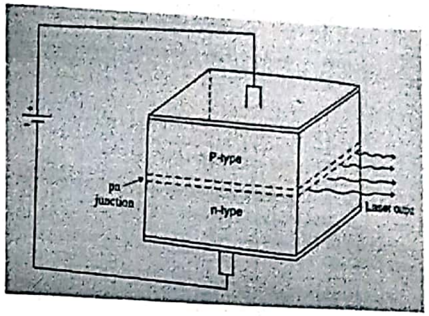
- The diode laser is made up of same type of semiconducting material on both sides of the junction is called Homo junction Semiconductor diode laser.
- Example : GaAs (Gallium Arsenide)
- It is specially fabricated p-n junction diode and it is forward biased.

Principle:

- When p-n junction is forward biased, the electrons from n region and holes from p region recombine with each other at the junction.
- During recombination process, light of photon is emitted.

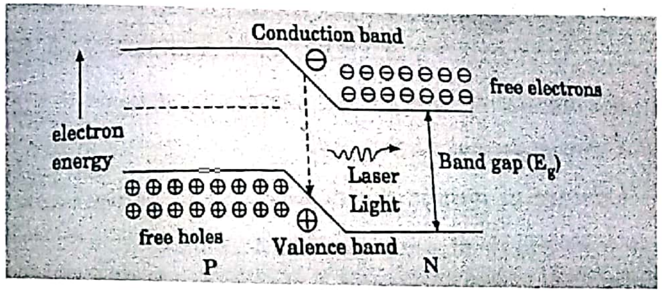
Construction:

- The active medium (p-n junction diode) is made up of single crystal of gallium arsenide.
- This crystal is cut into platelet having thickness 0.5mm.
- The platelet consists of n region and p region.
- The metal electrodes are connected to upper and lower surface and forward bias voltage applied through the metal electrodes.
- The end faces of p-n junction are well polished and it act as optical resonator through which laser light is emitted.



Working:

- When the p-n junction is forward biased, the electrons and holes are injected into junction region.
- The region around the junction contains large number of electrons in conduction band and holes in valence band.
- Now the electrons and hole recombines with each other.
- During recombination process, light of photons are emitted.
- When the forward bias voltage is increased, more photons are emitted.
- These photons move at the plane of the junction travel back and forth between two polished surfaces of the junction.
- After enough strength is attained, a laser beam of wavelength 8400Å is emitted from the junction.



Advantages:

- Very small size and compact
- High efficiency
- It is operated with less power
- Requires very little additional equipment
- It emits continuous wave output or pulsed output.

Disadvantages:

- Large divergence
- Purity and monochromaticity are poor
- Poor coherence and stability

Applications:

- Used in fiber optic communication
- Used in laser printers and CD players
- Used to heal the wounds
- Used as a pain killer

Hetero junction Semiconductor diode laser:

Definition:

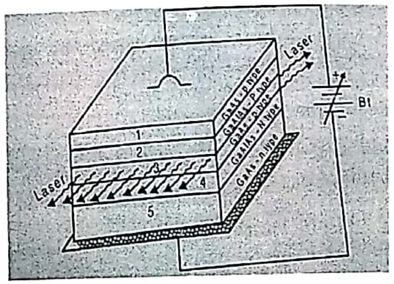
- The diode laser is made up of different type of semiconducting materials in two regions n type and p type is called Homo junction Semiconductor diode laser.
- Example : GaAs (Gallium Arsenide) and GaAlAs (Gallium Aluminium Arsenide)
- It is specially fabricated p-n junction diode and it is forward biased.

Principle:

- When p-n junction is forward biased, the electrons from n region and holes from p re recombine with each other at the junction.
- During recombination process, light of photon is emitted.

Construction:

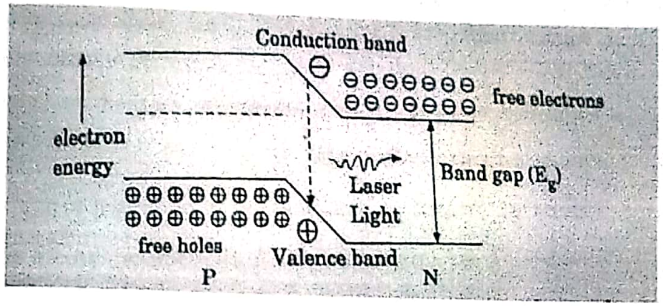
- It consists of five layers.
- Third layer (GaAs p-type) act as an active medium.
- It is kept in between Second layer (GaAlAs p-type) and Fourth layer (GaAlAs n-type)
- The metal electrodes are connected to upper and lower surface and forward bias volta applied through the metal electrodes.
- The end faces of p-n junction (Third and Fourth layer) is well polished and it act as op resonator through which laser light is emitted.



Working:

- When the p-n junction is forward biased, the electrons and holes are injected into jun region.
- The region around the junction contains large number of electrons in conduction banc holes in valence band.
- Now the electrons and hole recombines with each other.
- During recombination process, light of photons are emitted.
- When the forward bias voltage is increased, more photons are emitted.

- These photons move at the plane of the junction travel back and forth between two polished surfaces of the junction.
- After enough strength is attained, a laser beam of wavelength 8000Å is emitted from the junction.



Advantages:

- Produces continuous wave output
- Output power is very high

Disadvantages:

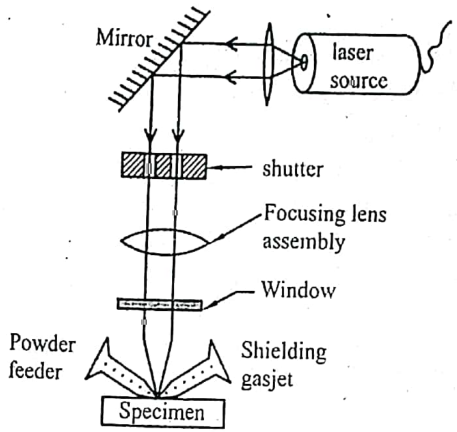
- High cost
- It is difficult to grow different layers of p-n junction

Applications:

- Used in optical communication
- Used in computers, especially CD-ROM.

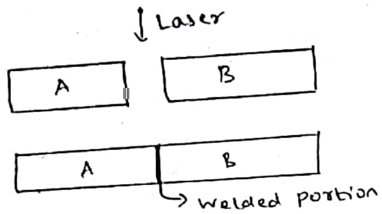
APPLICATION OF LASER (MATERIAL PROCESSING):

- A laser setup used for welding, cutting, drilling is shown in figure.
- The light output from laser source is incident on a plane mirror.
- After reflection, the laser light passes through the shutter to control the intensity of laser.
- Then there is a focusing lens assembly to get a fine beam.
- Further there is a shielding gas jet and powder feeder.
- The shielding gas jet is used to remove the molten materials.
- The powder feeder is used to spray the metal powder on the substrate.
- Using high power lasers, copper and aluminium can be welded and drilled.



Laser Welding:

- Welding means joining of two or more metal pieces into single unit.
- If laser beam is focused on a particular area for longer time, heating effect will be produced. This is called Thermal effect.
- As shown in figure, Let the two ends (A & B) of the specimen are brought together.
- Now the laser beam is focused for a long time on the ends that are to be joined.
- Due to thermal effect, portions that to be welded goes to molten liquid state.
- At this particular state, atoms in one end diffuse to the atoms in other end.
- Hence a perfect welding can be obtained.

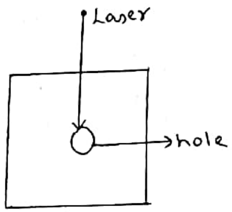


Advantages:

- It is a contactless process.
- It can be done even at very small places.
- Any dissimilar metals can be welded.

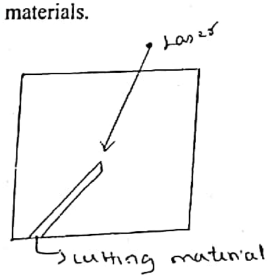
Laser drilling:

- The high intensity laser beam is made to fall on the material.
- Due to application of laser beam for long time, the temperature of the material is increased.
- Hence it is used to drilling holes.



Laser Cutting:

- The high intensity laser beam is made to fall on the material.
- Due to application of laser beam for long time, the temperature of the material is increased.
- Hence it is used to cutting materials.



Advantages:

- By high power laser like CO2 laser, glass and quartz are easily cut.
- It can be done at room temperature.
- Higher cutting speed can be achieved.

Principle and propagation of light in optical fiber:

Derivation of Acceptance angle and Numerical Aperture:

* Consider the light propagation in optical fiber.
* The incident ray AO enters into core at an angle of θ_0 to the fiber axis.
* It is refracted along OB at an angle of θ_r.
* This refracted ray falls on core-cladding interface at an angle of $90-\theta_r$ and it moves along BC.
* Let n_1, n_2 and n_0 be the refractive indices of core, cladding & air.

Applying Snell's law of refraction at the point of incidence of ray AO,

$$n_0 \sin\theta_0 = n_1 \sin\theta_r$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_r$$

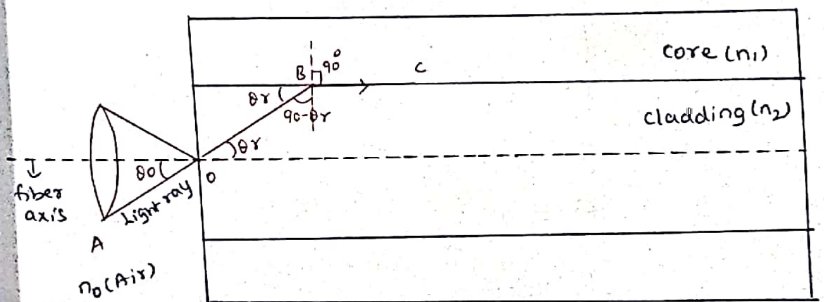
$$= \frac{n_1}{n_0}\sqrt{1-\cos^2\theta_r} \quad \text{--- (1)}$$

Applying Snell's law of refraction at the point B,

$$n_1 \sin(90-\theta_r) = n_2 \sin 90^\circ$$

$$n_1 \cos\theta_r = n_2$$

$$\cos\theta_r = \frac{n_2}{n_1} \quad \text{--- (2)}$$



Sub eqn ② in ①

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$= \frac{n_1}{n_0}\sqrt{\frac{n_1^2-n_2^2}{n_1^2}}$$

$$= \frac{n_1}{n_0 \times n_1}\sqrt{n_1^2-n_2^2}$$

$$= \frac{\sqrt{n_1^2-n_2^2}}{n_0}$$

For air, $n_0 = 1$

$$\therefore \sin\theta_0 = \sqrt{n_1^2-n_2^2}$$

$$\boxed{\theta_0 = \sin^{-1}\sqrt{n_1^2-n_2^2}}$$

Acceptance Angle :

The maximum angle at which the light can [...] total internal reflection is called acceptance angle (θ_0 [...]

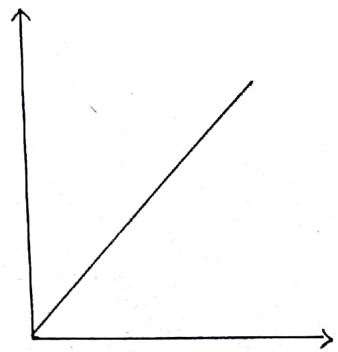
$$\boxed{\theta_0 = \sin^{-1}\sqrt{n_1^2-n_2^2}}$$

Numerical Apesture :

It is defined as the sine of acceptance [...]

$$NA = \sin\theta_0$$

$$\boxed{NA = \sqrt{n_1^2-n_2^2}}$$



Fractional Index change (Δ):

It is the ratio between refractive index difference between core + cladding to the refractive index of core.

$$\Delta = \frac{n_1-n_2}{n_1}$$

Relation between NA + Δ :

$$n_1\Delta = n_1-n_2$$

W.K.T $\quad NA = \sqrt{n_1^2-n_2^2}$

$$= \sqrt{(n_1+n_2)(n_1-n_2)}$$

$$= \sqrt{(n_1+n_2)\,n_1\Delta}$$

$n_1 \simeq n_2 \quad = \sqrt{(n_1+n_1)n_1\Delta}$

$$= \sqrt{2n_1^2\Delta}$$

$$\boxed{NA = n_1\sqrt{2\Delta}}$$

TYPES OF OPTICAL FIBER:

- Optical fibers are classified into different types based on
  - ✓ Material
  - ✓ Number of modes
  - ✓ Refractive index profile

(a) Classification based on material:

- Based on material, it is divided into two types.
  - ✓ Glass fiber
  - ✓ Plastic fiber

(i)Glass fiber:

- The optical fiber is made up of mixture of silica glasses and metal oxides.
- Example: Core – GeO_2-SiO_2, Cladding – SiO_2.

(ii)Plastic fiber:

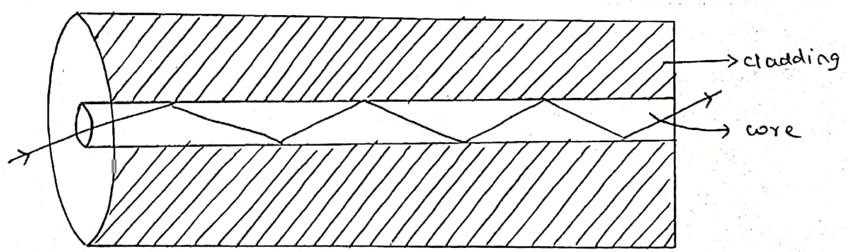
- The optical fiber is made up of plastics.
- Example: Core –Polystyrene, Cladding – Methyl methacrylate.
- Advantages: It is very cheap and flexible and it can be handles without any special care.

(b) Classification based on number of modes:

- Based on number of modes, it is divided into two types.
  - ✓ Single mode fiber
  - ✓ Multimode fiber

(i)Single mode fiber:

- Only one mode is transmitted through the optical fiber.



Characteristics:

- It can support only one mode of propagation.
- The core diameter is small.
- They have small refractive index difference of core and cladding.

Applications:

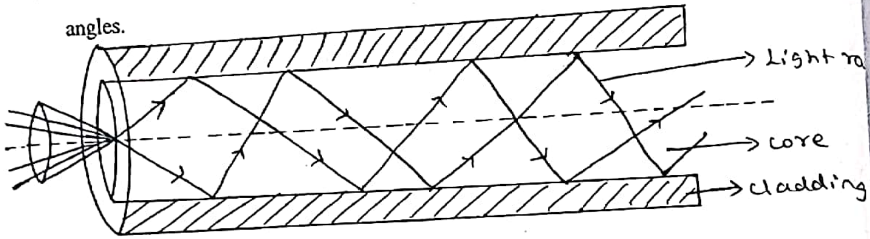
- It is used in laser diodes.

(ii)Multimode fiber:

- More than one mode is transmitted through an optical fiber.

Characteristics:

- It can support many modes of propagation.
- The core diameter is large and makes easier to launch the fiber through various angles.



(C)Refractive index profile:

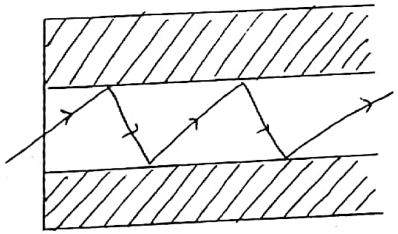
- The variation of refractive index with respect to radial distance from the fiber axis.
- Based on the variation in refractive index of core and cladding, it is divided into two types.
  - ✓ Step index fiber
  - ✓ Graded index fiber

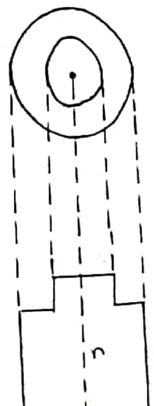
(i)Step index fiber:

- The variation in refractive indices of core and cladding vary step by step.
- It is further divided into two types.
  - ✓ Step index single mode fiber
  - ✓ Step index multimode fiber

Step index single mode fiber:

- The core diameter has 5 to 10 μm and the cladding diameter has 50 to 125 μm.
- The refractive index changes in step at core cladding boundary.
- Due to small core diameter, only single mode can be transmitted.





Characteristics:

- The core diameter is thin.
- Numerical aperture is very small.
- It can support one mode of propagation.
- There is no signal loss.
- It has high bandwidth

Advantages:

- It has very high rate of data transmission.
- About 80% of the optical fiber is manufactured by this method

Disadvantages:

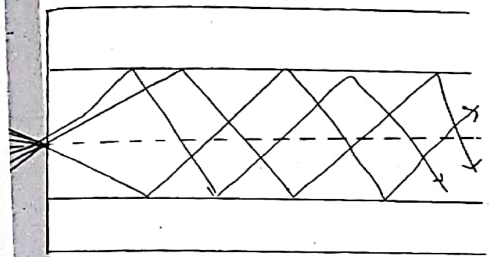
- High cost

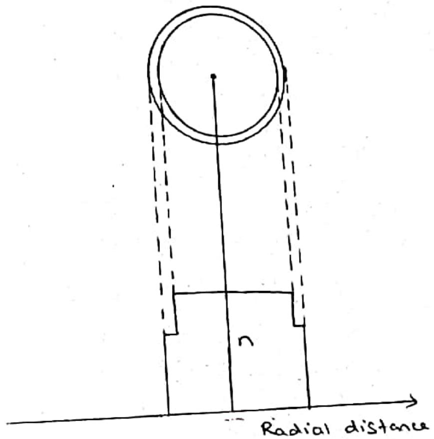
Applications:

- It is used in undersea cable for long distance communication.
- It is used in submarine cable system.

Step index multimode fiber:

- The core diameter has 50 to 200 μm and the cladding diameter has 125 to 300 μm.
- The refractive index changes in step at core cladding boundary.
- Due to large core diameter, many modes mode can be transmitted.





Characteristics:

- Large core diameter
- Low bandwidth
- Large numerical aperture

Advantages:

- Easy to operate
- Longer life
- Low cost
- Easy to couple the fiber

Disadvantages:

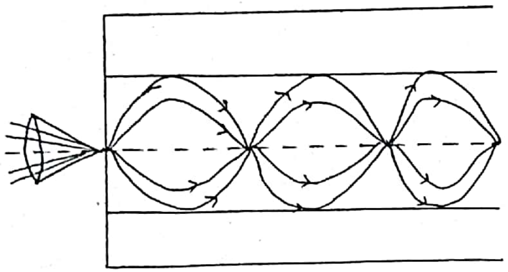
- Data transmission is less efficient.
- They suffer intermodal dispersion loss.

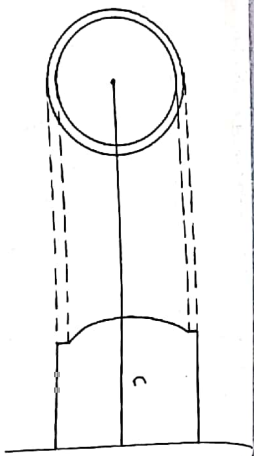
Applications:

- It is used in data links.

(ii)Graded index fiber (multimode):

- The core diameter has 50 to 200 μm and the cladding diameter has 100 to 250 μm.
- The refractive index of core is maximum at the axis of fiber and gradually decreases towards the cladding.





Characteristics:

- Small numerical aperture
- Intermediate bandwidth
- The source of light is LED or Laser.

Advantages:

- High quality fiber
- Intermodal dispersion loss is reduced

Disadvantages:

- High cost
- Fabrication is difficult
- Coupling of fiber is difficult

Applications:

- It is used in intra city trunks between central telephone offices.
- It is used in medium distance applications.

FIBER OPTICS

LOSSES IN OPTICAL FIBERS:

- When light propagates through an optical fiber, a small percentage of light is lost through different mechanism.
- The loss of optical fiber is measured in decibel/kilometer for attenuation loss

Attenuation:

It is defined as the ratio of optical power output to the optical power input from a fiber of length 'L'

$$\text{Attenuation } (\alpha) = -\frac{10}{L} Log \frac{\text{Output Power}}{\text{Input Power}} \; dB/Km$$

Types of Attenuation Loss:

- ✓ Absorption loss
- ✓ Scattering loss
- ✓ Radiative loss

Absorption Loss:

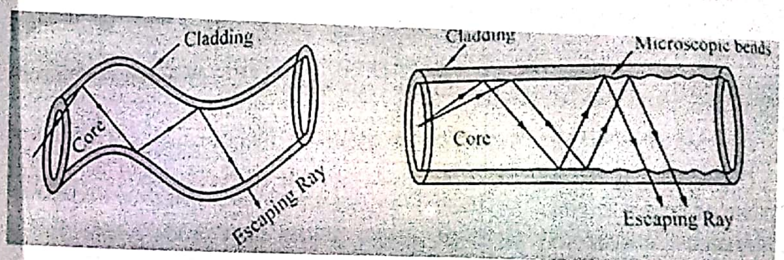
- It occurs due to imperfection of atomic structures such as missing molecules.
- It also occurs due to presence of impurities in the fiber material.

Scattering Loss:

- It is a wavelength dependent loss.
- If the light is passed through the fiber, a portion of light is scattered.
- This type of scattering is called Rayleigh scattering.

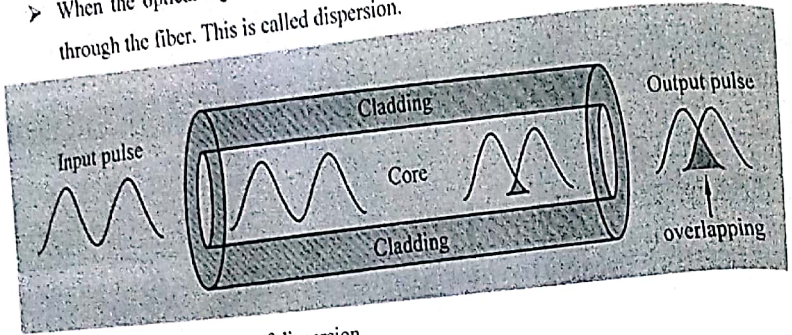
Radiative Loss:

- Due to bending of fiber, it is classified into two types.
  - ✓ Macroscopic bends – In this type the whole fiber will be bend and hence the light ray escapes from the fiber.
  - ✓ Microscopic bends – It is due to micro bending inside the fiber and this appears during manufacturing.



Distortion and Dispersion:

- When the optical signal is sent into the fiber, the signal broadens as it propagates through the fiber. This is called dispersion.



- There are three types of dispersion.
  - ✓ Intermodal dispersion
  - ✓ Material/Chromatic dispersion
  - ✓ Waveguide dispersion

Intermodal dispersion:

- When more than one mode is propagating through the fiber, this dispersion will occur.
- Since many modes are propagating, they have different wavelength and will take different time to propagate through the fiber.

Material/Chromatic dispersion:

- This dispersion occurs due to different wavelength of light travelling at different speed inside the fiber.

Waveguide dispersion:

- This dispersion arises due to different angles at which they incident on core-cladding interface.

FIBER OPTIC SENSORS:

It is a transducer which converts any form of signal to optical signal. Here optical fiber is used as a guiding medium.

Types of Sensors:

- Intrinsic sensors (or) Active sensors
- Extrinsic sensors (or) Passive sensors

Intrinsic Sensors (or) Active Sensors:

- The physical parameter are sensed directly act on the fiber.
- Example: Pressure/Temperature Sensor

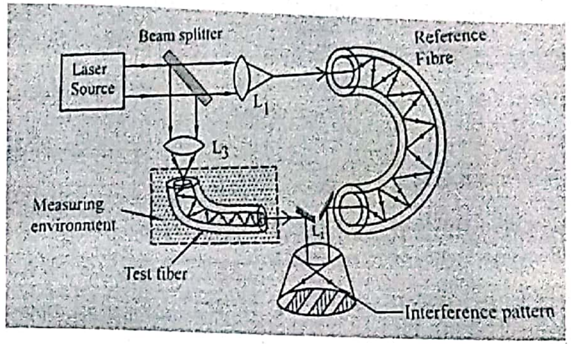
Pressure/Temperature Sensor:

Principle:

It is based on the principle of Interference between the beams emerging from reference fiber and test fiber.

Description:

- It consists of laser source to emit laser.
- A beam splitter made of glass is inclined at an angle of 45^0 is placed near to laser beam.
- Two fibers are placed (via) Reference fiber is isolated from the environment and the Test fiber is kept in environment.
- Separate lens system is provided and collects the beam.



Working:

- A laser light is emitted from laser source.
- The beam splitter splits the beam into two (i) main beam and (ii) splitted beam exactly at right angles to each other.
- The main beam after passing through the reference fiber and it is made to falls on the lens L2.
- The splitted beam passes through the lens L3 and it is focused on the test fiber and it is made to falls on the lens L2.
- The two beams after passing through the fiber produces a path difference due to change in pressure and temperature.
- Due to path difference the interference pattern can be measured.
- Therefore change in pressure or temperature can be accurately measured with the help of interference pattern.

Extrinsic Sensors (or) Passive Sensors:

- In this sensor, separate sensing element is used.
- Example: Displacement Sensor

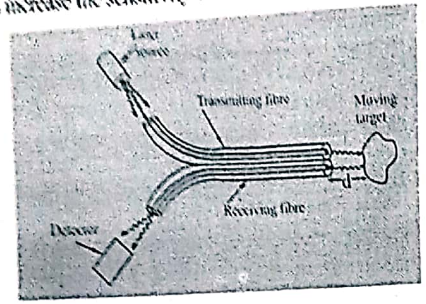
Principle:

- Light is sent through a transmitting fiber and is made to fall on the moving target.
- The reflected light from the target is sensed by a detector.

- With respect to the intensity of light received, the displacement of the target is measured.

Construction:

- It consists of bundle of transmitting fibers coupled to the laser source and the bundle of receiving fiber coupled to the detector.
- The axis of transmitting fiber and receiving fiber with respect to the moving target can be adjusted to increase the sensitivity of the sensor.



Working:

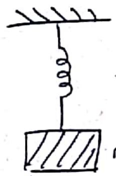
- Light from laser source is transmitted through the transmitting fiber and it is made to fall on the moving target.
- The light reflected from the target is made to pass through the receiving fiber and it is detected by detector.
- Based on intensity of light received, the displacement can be measured.
- If the received intensity is more, the target is moving towards the sensor.
- If the received intensity is less, the target is moving away from the sensor.

Damped oscillations:

The vibrations in air or other medium is damped. Some amount of energy is lost & it is overcoming the resistive force. Hence amplitude decreases with time & becomes zero. This is called damped oscillations.

Ex: Oscillations of simple pendulum.

Differential Equations & its solution for damped oscillations:



* consider a particle of mass m attached to a spring executes simple harmonic motion.

* Let y be the displacement at any time t.

The restoring force $\propto -y$

$$f_1 = -ky$$

The frictional or damping force $\propto -v$

$$f_2 = -r\frac{dy}{dt}$$

∴ The total force acting on the body is

$$f = f_1 + f_2$$

$$f = -ky - r\frac{dy}{dt} \quad \text{—— (1)}$$

From Newton's II law

$$f = m\frac{d^2y}{dt^2} \quad \text{—— (2)}$$

From eqn (1) & (2)

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt}$$

$m \frac{d^2y}{dt^2} + r \frac{dy}{dt} + ky$

$\div$ by m $\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m} y = 0.$

Sub $\frac{r}{m} = 2b$; $\frac{k}{m} = w^2$

$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + w^2 y = 0$ —— ③

This is the differential equation of damped harmonic oscillator.

The solution of eqn ③ is

$y = Ae^{\alpha t}$ —— ④

Differentiating w.r.to 't'

$\frac{dy}{dt} = A\alpha e^{\alpha t}$ —— ⑤

$\frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t}$ —— ⑥

Sub eqn ④, ⑤, ⑥ in eqn ③

$\therefore A\alpha^2 e^{\alpha t} + 2bA\alpha e^{\alpha t} + w^2 Ae^{\alpha t} = 0.$

$Ae^{\alpha t}[\alpha^2 + 2b\alpha + w^2] = 0$

$Ae^{\alpha t} \neq 0 \quad \alpha^2 + 2b\alpha + w^2 = 0$

$\alpha = \frac{-2b \pm \sqrt{4b^2 - 4w^2}}{2}$

$= \frac{-2b \pm 2\sqrt{b^2-w^2}}{2}$

$\boxed{\alpha = -b \pm \sqrt{b^2-w^2}}$

$\therefore$ The general solution is

$y = A_1 e^{(-b+\sqrt{b^2-w^2})t} + A_2 e^{(-b-\sqrt{b^2-w^2})t}$

Special

Case (i) Heavy damping (or) dead beat

when $b^2 > w^2$

$\sqrt{b^2-w^2}$ is real & less than b.

$\therefore -b+\sqrt{b^2-w^2}$ & $-b-\sqrt{b^2-w^2}$ are negative.

The exponential term is zero & there is no oscillation.

Ex: oscillation of pendulum in oil.

Case (ii) critical damping

when $b^2 = w^2$; $b^2 - w^2 = 0$

Sub this in eqn ⑦

$\therefore y = (A_1 + A_2) e^{-bt}$

It is not a proper solution of eqn ③

So we can take $b^2 - w^2 = h$

Eqn ⑦ $\Rightarrow \therefore y = A_1 e^{(-b+h)t} + A_2 e^{(-b-h)t}$

$= A_1 e^{-bt} e^{ht} + A_2 e^{-bt} e^{-ht}$

$= e^{-bt}[A_1 e^{ht} + A_2 e^{-ht}]$

$= e^{-bt}[A_1(1+ht) + A_2(1-ht)]$

$= e^{-bt}[A_1 + A_1 ht + A_2 - A_2 ht]$

$= e^{-bt}[(A_1+A_2) + ht(A_1-A_2)]$

Sub $A_1 + A_2 = p$ & $h(A_1 - A_2) = q$

$\therefore y = e^{-bt}(p+qt)$

When t increases, p+qt increases & e^{-bt} decr

Ex: voltmeter, ammeter.

case (iii) : Low damping

$b^2 < \omega^2$

$\sqrt{b^2-\omega^2}$ is imaginary & negative.

$\therefore \sqrt{b^2-\omega^2} = i\sqrt{\omega^2-b^2} = i\beta.$

Eqn ⑦ $\Rightarrow$ $y = A_1 e^{(-b+i\beta)t} + A_2 e^{(-b-i\beta)t}$

$= A_1 e^{-bt} e^{i\beta t} + A_2 e^{-bt} e^{-i\beta t}$

$= e^{-bt} [A_1 e^{i\beta t} + A_2 e^{-i\beta t}]$

$= e^{-bt} [A_1 (\cos\beta t + i\sin\beta t) + A_2(\cos\beta t - i\sin$

$= e^{-bt} [A_1 \cos\beta t + A_1 i \sin\beta t + A_2 \cos\beta t -$

$= e^{-bt} [(A_1+A_2)\cos\beta t + (A_1-A_2) i\sin\beta t]$

$= e^{-bt} [a\sin\phi\cos\beta t + a\cos\phi\sin\beta t]$

$= a e^{-bt} [\sin(\beta t+\phi)]$

$y = a e^{-bt} \{\sin[(\sqrt{\omega^2-b^2})t+\phi]\}$

e^{-bt} decreases with time & angular frequency

Ex: Electric oscillations of L-C-R circuit.

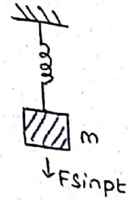
The oscillating particle oscillated with a frequency other than natural frequency is called forced oscillation.

Ex: vibrating tuning fork on table.

Differential Equation & its solution for forced oscillations:

* Consider a particle of mass 'm' connected to a spring.

* Hence three forces will act on the particle.



(i) Restoring force $\propto -y$

$F_1 = -ky$

(ii) Frictional force $\propto -v$

$F_2 = -r\frac{dy}{dt}$

(iii) External periodic force $F_3 = F\sin pt$

$\therefore$ The net force acting on the particle is

$$F = -ky - r\frac{dy}{dt} + F\sin pt \quad \text{— ①}$$

From Newton's second law,

$$F = m\frac{d^2y}{dt^2} \quad \text{— ②}$$

From ① & ②

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + F\sin pt$$

$\div$ by m

$$m\frac{d^2y}{dt^2} + \frac{k}{m}y + \frac{r}{m}\frac{dy}{dt} = \frac{F}{m}\sin pt$$

put $\frac{r}{m} = 2b$, $\frac{k}{m} = \omega^2$ & $\frac{F}{m} = f$

$\therefore \frac{d^2y}{dt^2} + 2b \frac{dy}{dt} ...$

This is the differential equation of forced oscillation.

The solution of eqn ③ is

$$y = A \sin(pt - \theta) \quad \text{--- ④}$$

where $A \rightarrow$ amplitude.

$\theta \rightarrow$ angle.

Differentiating eqn ④ w.r.to 't'

$$\frac{dy}{dt} = Ap \cos(pt - \theta) \quad \text{--- ⑤}$$

Differentiating again w.r.to 't'

$$\frac{d^2y}{dt^2} = -Ap^2 \sin(pt - \theta) \quad \text{--- ⑥}$$

Sub eqn ④, ⑤, ⑥ in eqn ③

$$-Ap^2 \sin(pt-\theta) + 2b\, Ap \cos(pt-\theta) + A\omega^2 \sin(pt-\theta) = f \sin pt$$

$$A(\omega^2 - p^2) \sin(pt-\theta) + 2bAp \cos(pt-\theta) = f[\sin(pt-\theta) ...$$

$$A(\omega^2 - p^2) \sin(pt-\theta) + 2bAp \cos(pt-\theta) = f[\sin(pt-\theta) ... + \cos(pt-\theta) \sin\theta]$$

Equating the coefficient of $\sin(pt-\theta)$ & $\cos(pt-\theta)$ both sides

$$A(\omega^2 - p^2) = f\cos\theta \quad \text{--- ⑦}$$

$$2bAp = f\sin\theta \quad \text{--- ⑧}$$

$⑦^2 + ⑧^2 \Rightarrow A^2(\omega^2 - p^2)^2 + 4b^2A^2p^2 = f^2(\cos^2\theta + \sin^2\theta)$

$$A^2(\omega^2 - p^2)^2 + 4b^2A^2p^2 = f^2$$

$$A^2[(\omega^2 - p^2)^2 + 4b^2p^2] = f^2$$

$$A^2 = \frac{f^2}{(\omega^2 - p^2)^2 + 4b^2p^2}$$

$$\boxed{A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}} \quad \text{--- ⑨}$$

$$\frac{\text{Eqn ⑧}}{\text{Eqn ⑦}} \Rightarrow \tan\theta = \frac{2bAp}{A(\omega^2 - p^2)} = \frac{2bp}{\omega^2 - p^2}$$

$$\therefore \boxed{\theta = \tan^{-1} \frac{2bp}{\omega^2 - p^2}} \quad \text{--- ⑩}$$

The eqn ⑨ & ⑩ gives the amplitude & phase of forced vibration.

The amplitude & phase will depend on driving frequency (p) & natural frequency of the oscillator (ω).

<u>Case (i)</u>: $p \ll \omega$, $p \approx 0$

$$A = \frac{f}{\sqrt{(\omega^2)^2 + 0}} = \frac{f}{\omega^2}$$

$$\theta = \tan^{-1}(0) = 0.$$

Hence the amplitude of vibration is independent of frequency of force.

<u>Case (ii)</u> $p = \omega$

$$A = \frac{f}{\sqrt{0 + 4b^2\omega^2}} = \frac{f}{2b\omega} = \frac{F \times m}{m \times r \times \omega}$$

$$= \frac{F}{r\omega}$$

$= \tan^{-1}(\alpha) = \pi/2$

Displacement lags behind the force by $\pi/2$

Case (iii) $p >> \omega$, $\omega = 0$

$$A = \frac{f}{\sqrt{p^4}}$$

$$= \frac{f}{p^2} = \frac{F}{mp^2}$$

$$\theta = \tan^{-1}\left(\frac{2b\cancel{p}}{-p^{\cancel{2}}}\right)$$

$$= \tan^{-1}(-0)$$

$$\theta = \pi$$

Displacement lags behind the force by π.

Plane ...

If the sphere of large thickness, the spherical wave is approximate to plane surface & it is called plane progressive wave.

Relation between frequency, wave speed & wavelength:

Distance travelled in one second $= \frac{\lambda}{T} = v$

$$\therefore \boxed{\lambda = vT} \quad \text{--- (1)}$$

We know that $T = \frac{1}{n}$

$$\lambda = \frac{v}{n}$$

$$\boxed{\therefore v = n\lambda} \quad \text{--- (2)}$$

Wave Equation of a Plane Progressive wave:

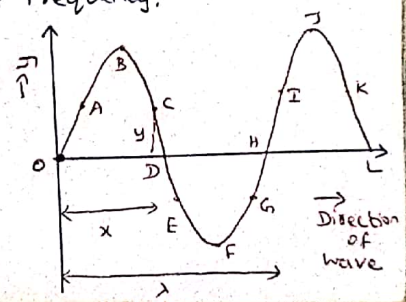
* Suppose a plane progressive wave is propagating in a medium along x-axis.
* The particle execute simple harmonic motion.
* The position of the particles O, A, B, C, D, ... are noted.
* As the wave propagates, all the particles are vibrate along the mean position.

$\therefore$ The displacement of the particle at time t is

$$y = A \sin \omega t \quad \text{--- (1)}$$

where A → Amplitude, ω → Angular frequency.

If v is the speed of wave & c is a particle at a distance of x from O, then the time taken is $\frac{x}{v}$.



$\therefore \; t \to t - x/v$ (3)

$$y = A \sin \omega (t - x/v) \quad \text{--- (2)}$$

$$\omega = \frac{2\pi}{T}$$

$$y = A \sin \left(\frac{2\pi}{T}\right) (t - x/v)$$

$$= A \sin 2\pi \left(\frac{t}{T} - \frac{x}{vT}\right)$$

$$= A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right] \quad [\because \lambda = vT]$$

$$= A \sin \frac{2\pi}{\lambda} \left[\frac{\lambda t}{T} - x\right]$$

$$y = A \sin \frac{2\pi}{\lambda} [vt - x] \quad [\because v = \lambda/T] \quad \text{--- (3)}$$

$$= A \sin \left[\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right]$$

$$= A \sin \left[\frac{\omega \cancel{v} t}{\cancel{v}} - \frac{\omega}{v} x\right]$$

$$\boxed{y = A \sin (\omega t - kx)} \quad [\because \frac{\omega}{v} = k] \quad \text{--- (4)}$$

If the wave propagating along Negative x ax

$$y = A \sin (\omega t + kx)$$

The equation of wave is

$$\boxed{y = A \sin[(\omega t - kx) + \phi]} \quad \text{--- (5)}$$

where $\phi \to$ phase difference.

The eqn (5) is the most general equati of plane progressive wave travelling along positive x-axis.

Differential Equation of wave motion:

From eqn (3) (32)

$$y = A \sin \frac{2\pi}{\lambda} (vt - x)$$

Differentrate w.r.to 't'

$$\frac{dy}{dt} = A \times \frac{2\pi v}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x)$$

$$\frac{d^2y}{dt^2} = -A \left[\frac{+2\pi v}{\lambda}\right]^2 \sin \frac{2\pi}{\lambda} (vt - x)$$

$$\frac{1}{v^2} \frac{d^2y}{dt^2} = -A \left(\frac{2\pi}{\lambda}\right)^2 \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{--- (6)}$$

Differentiate w.r.to 'x'

$$\frac{dy}{dx} = A \left(-\frac{2\pi}{\lambda}\right) \cos \frac{2\pi}{\lambda} (vt - x)$$

$$\frac{d^2y}{dx^2} = -A \left(-\frac{2\pi}{\lambda}\right)^2 \sin \frac{2\pi}{\lambda} (vt - x) \quad \text{--- (7)}$$

Equating (6) & (7)

$$\frac{1}{v^2} \frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$$

$$\boxed{\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}} \quad \text{--- (8)}$$

This is the differential equation of wave motion

For plane progressive wave, $A = a$, $t \to t + \delta t$, $x \to x + v\delta t$

Eqn (3) $\Rightarrow$ $y' = a \sin \frac{2\pi}{\lambda} [v(t + \delta t) - (x + v\delta t)]$

$$= a \sin \frac{2\pi}{\lambda} [vt + \cancel{v\delta t} - x - \cancel{v\delta t}]$$

$$= a \sin \frac{2\pi}{\lambda} (vt - x)$$

$$= y.$$

UNIT-3 Thermal physics

→ Mechanical Engg — design of Internal & external combustion engine, refrigeration, A.C etc.

→ Electrical Engg - design of cooling system for motor, transformer & generator

→ Civil Engg - control of heat transfer in dams, buildings etc.

→ Chemical Engg. - app. of heat trasfer in freezing, boiling, evaporation & condensation process.

Thermal expansion of solids :-

The ratio b/n change in dimension to original dimension per unit rise in temp.

Types

→ linear expansion (Expansion in length)

Coeff of linear expansion of solid is the ratio b/n increase in length per unit length when its temp is raised to 1K . α.

$$\alpha = \frac{L_2 - L_1}{L_1(T_2 - T_1)}$$

→ Superficial expansion (β). → Expansion in area.

$$\beta = \frac{A_2 - A_1}{A_1(T_2 - T_1)}$$

→ cubical expansion (γ) – expansion in volume

$$\gamma = \frac{V_2 - V_1}{V_1 (T_2 - T_1)}$$

unit → per °C (or) per K.

Thermal expansion of liquid. (volume).

liquid → container → heat → liquid + container. → ignore the expansion of container

we get apparent expansion.

→ coefficient of apparent expansion of a liquid :-

It is the ~~ratio of~~ observed ↑se in volume of liquid per unit original volume ~~of liquid~~ per ~~degree~~ kelvin rise in temp.

$$\gamma_a = \frac{V_2 - V_1}{V_1 (T_2 - T_1)}$$

→ coefficient of real or absolute expansion of liquid :-

It is the real ↑se in volume of liquid per unit original volume ~~of liquid~~ per ~~degree~~ kelvin rise in temp.

$$\gamma_r = \frac{V_2 - V_1}{V_2 (T_2 - T_1)}$$

Application of expansion of solids :-

* Expansion joints & bimetallic strip.

while constructing a large area of beam, gaps are produced & these gaps are called expansion joints.

Types: → Metallic E.J
→ wall
→ slip type
→ fabric
→ Rubber
→ gimbal
→ pressure balance.

Metal Expansion joints

→ also called as compensator.
→ in pipeline, container & machine.

Types of movements:
- axial E.J → T. expansion of st. line section b/n 2 fixed pts
- angular → It requires two or three joints
  - two → two hinge
  - three → three hinge system
- lateral. → 90° within single plane or multiplane.

App

→ Heating and air conditioning installation.
→ Hot water + fire protection system
→ refrigeration circuits.

Wall expansion joints

expansion of concrete due to temp changes.

→ It is a mid structure separation designed to relieve stress caused by temp changes, wind, seismic events etc.

→ bridges made up of steel - having long life.

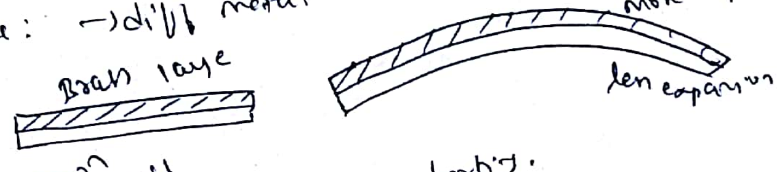
→ without these joints, it would crash.

Bimetallic strip

A strip is made of 2 metals of different expansion coefficient joined together.

It is like a compound bar.

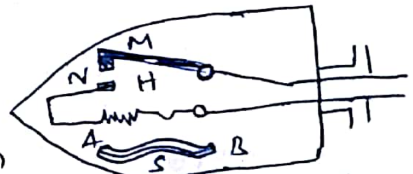
Principle: → diff metal has diff coefficient of expansion.



App :-

(i) Bimetallic thermostat :-

- To maintain constant temp.
- M - contact maker
- Temp ↑, expansion of brass causes the spring unwind & breaking the circuit
- Temp ↓ contraction of brass to bend in opp. direction to make the spring wind & make the electric circuits.



(ii) Bimetallic thermometer:

- Bimetallic strip - form of spring.
- → Temp ↑, the strip bend & the pointer moves.

Adv :-

- simple & inexpensive
- accuracy + or - 2% to 5%
- withstand 50% over range of temp.

Limitations

- temp not more than 400°C
- regular used, introduce error.

(iii) Bimetallic automatic fire alarm: electric bell. → ringing.

(iv) Bimetallic sensor

convert temp change to mechanical displacement.

Transfer of Heat energy

→ conduction

→ convection

→ Radiation.

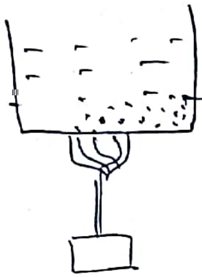
→ Thermal conduction :-

It is the process in which heat is transferred from one point to another without actual motion of particals.

→ Thermal convection :-

" "

transmitted from one place to another ~~with~~ by actual motion of particles.



Convection - type <
- natural → due to temp diff within the fluid.
- forced → caused by external mechanism by a pump, fan

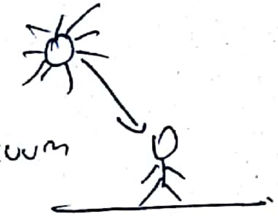
The Amount of heat transfer by convection is calculated based of newton's law of cooling

$$Q = hA\Delta t$$

h → ~~Area~~ convective heat transfer coefficient
A → Area
Δt → temp diff.

Thermal radiation

It is the process in which heat is transmitted from one place to another place without any agency.



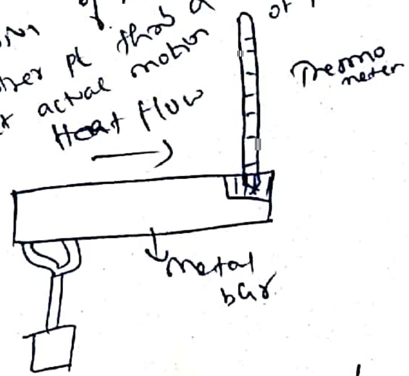
properties of Thermal radiation.

* They can travel through vacuum
* They travel in st. line
* They travel with same velocity as light
* It follows inverse square law.
* They exhibit the phenomenon of reflection & refraction

Heat conduction in solids

Defn. → process of transmission of heat from one point to another pt through a solid substance without actual motion of particles

The transmission of heat takes place ~~flows~~ by molecular vibration.



Thermal conductivity :

The ability of a substance to conduct heat energy.

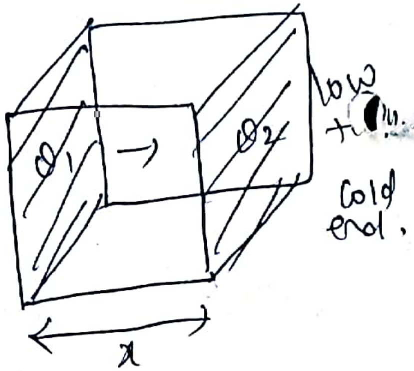
Expression for T.C

Amount of heat conducted $\propto A$

$\propto \theta_1 - \theta_2$

$\propto t$

$\propto \frac{1}{x}$.



$$\therefore Q \propto \frac{A(\theta_1 - \theta_2)t}{x}$$

$$= \frac{kA(\theta_1 - \theta_2)t}{x}$$

$$\boxed{k = \frac{Qx}{A(\theta_1 - \theta_2)t}}$$

Coefficient of thermal conductivity.

It is defined as the amount of heat conducted per ~~second~~ unit area per unit time per unit temp diff per unit length.

$\boxed{k = Q}$ $A = 1m^2$, $\theta_1 - \theta_2 = 1K$, $t = 1s$, $x = 1m$

$\frac{\theta_1 - \theta_2}{x}$ → rate of fall of temp w.r.to distance

temp. gradient $= \frac{d\theta}{dx}$

$$\therefore Q = -kA\frac{d\theta}{dx}t.$$

−ve sign indicates temp ↓ with distance.

Unit:

$$k = \frac{\text{Joule} \times m}{m^2 \times K \times s}$$

$$k = W m^{-1} K^{-1}$$

high k → copper 385

low k → card board 0.04.

Flow of heat through a compound media

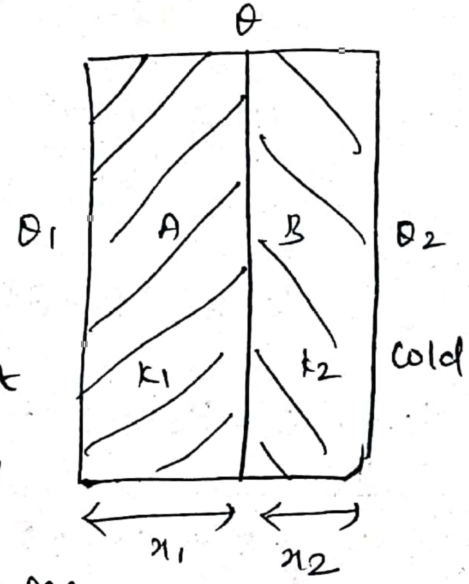
Material in series

A & B

k_1 & k_2

x_1 & x_2

A



Steady state reached

↓

Q → every layer is same

After the steady state is reached,

Amount of heat flowing thro' the material A per second is

$$Q = \frac{K_1 A (\theta_1 - \theta)}{x_1} \quad \text{—} (1)$$

" " B

$$Q = \frac{K_2 A (\theta - \theta_2)}{x_2} \quad \text{—} (2)$$

In steady state condition

eqn (1) = eqn (2)

$$\frac{K_1 A (\theta_1 - \theta)}{x_1} = \frac{K_2 A (\theta - \theta_2)}{x_2}$$

$$K_1 x_2 (\theta_1 - \theta) = K_2 x_1 (\theta - \theta_2)$$

$$K_1 x_2 \theta_1 - K_1 x_2 \theta = K_2 x_1 \theta - K_2 x_1 \theta_2$$

$$K_2 x_1 \theta + K_1 x_2 \theta = K_1 x_2 \theta_1 + K_2 x_1 \theta_2$$

$$\theta (K_2 x_1 + K_1 x_2) = K_1 x_2 \theta_1 + K_2 \theta_2 x_1$$

$$\theta = \frac{K_1 x_2 \theta_1 + K_2 x_1 \theta_2}{K_2 x_1 + K_1 x_2} \quad \text{—} (3)$$

Sub θ in eqn (1)

$$Q = \frac{K_1 A}{x_1} \left[\theta_1 - \frac{K_1 x_2 \theta_1 + K_2 x_1 \theta_2}{K_2 x_1 + K_1 x_2} \right]$$

$$= \frac{K_1 A}{x_1} \left[\frac{K_2 x_1 \theta_1 + K_1 x_2 \theta_1 - K_1 x_2 \theta_1 - K_2 x_1 \theta_2}{K_2 x_1 + K_1 x_2} \right]$$

$$= \frac{K_1 A K_2 x_1}{x_1} \left[\frac{\theta_1 - \theta_2}{K_2 x_1 + K_1 x_2} \right]$$

$$= \frac{K_1 K_2 A (\theta_1 - \theta_2)}{K_2 x_1 + K_1 x_2}$$

÷ by $K_1 K_2 A$

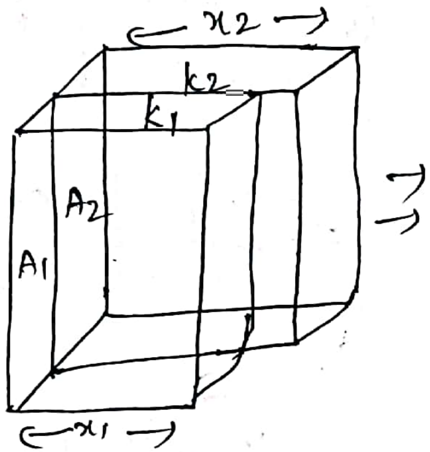
$$= \frac{A (\theta_1 - \theta_2)}{\frac{x_1}{K_1} + \frac{x_2}{K_2}} \quad \text{—} (4)$$

exp for amount of heat flowing thro' 2 materials.

More than 2 materials.

$$Q = \frac{A(\theta_1 - \theta_2)}{\sum x/k} \quad \text{--- (5)}$$

<u>Materials in parallel</u>



A & B

x_1 & x_2

k_1 & k_2

A_1 & A_2

Temp θ_1 & θ_2

$$A \Rightarrow Q_1 = \frac{k_1 A_1 (\theta_1 - \theta_2)}{x_1} \quad \text{--- (1)}$$

$$B \Rightarrow Q_2 = \frac{k_2 A_2 (\theta_1 - \theta_2)}{x_2} \quad \text{--- (2)}$$

$\therefore$ Total heat $= Q_1 + Q_2$

$$= (\theta_1 - \theta_2)\left[\frac{k_1 A_1}{x_1} + \frac{k_2 A_2}{x_2}\right] \quad \text{--- (3)}$$

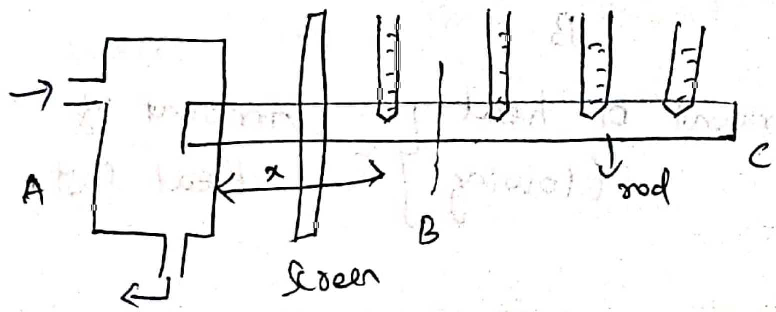
$$= (\theta_1 - \theta_2) \sum \frac{kA}{x} \quad \text{--- (4)}$$

Methods to determine Thermal conductivity

- → Searle's method - good conductor ex: metallic rods
- → Forbe's " - absolute conductivity of metals
- → Lee's disc " - bad conductor ex: cardboard
- → Radial flow " - "

<u>Forbe's Method.</u>

long rod heated at one end & steady state is reached after some time.



Amount of heat flowing per second at B is

$$= KA\left(\frac{d\theta}{dx}\right)_B \quad -①$$

$\left(\frac{d\theta}{dx}\right)_B \rightarrow$ temp gradient at B

This heat is equal to the heat lost by radiation by rod.

$\text{den} = \frac{\text{Mass}}{\text{volume}}$

Mass $= (A\,dx)\,\rho$

$dx \rightarrow$ thickness (assume)

$A\,dx \rightarrow$ volume

Heat lost = Mass × specific heat capacity × rate of fall of temp

$$= A\,dx\,\rho\,\frac{d\theta}{dt}\,S \quad -②$$

Total heat lost b/n B & C is

$$= \int_B^C A\,dx\,\rho\,S\,\frac{d\theta}{dt} \quad -③$$

Amount of heat flowing } = Amount of heat lost

$$K\cancel{A}\left(\frac{d\theta}{dx}\right)_B = \int_B^C \cancel{A}\,dx\,\rho\,S\,\frac{d\theta}{dt}$$

$$K = \frac{\rho S \int_B^C \frac{d\theta}{dt}\,dx}{\left(\frac{d\theta}{dx}\right)_B} \quad -④$$

Experiment consist of 2 parts

- Static experiment to find $\left(\frac{d\theta}{dx}\right)_B$
- Dynamic " to find $\frac{d\theta}{dt}$ & $\int_B^C \frac{d\theta}{dt}\,dx$.

(i) Static experiment

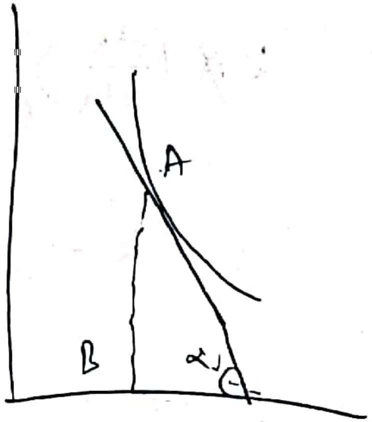
Thermometer record the temp at diff points.

When steady state reached

note θ & x ($x \rightarrow$ distance)

$$\left(\frac{d\theta}{dx}\right)_B = \frac{AB}{BC}$$

$$= \tan\alpha$$

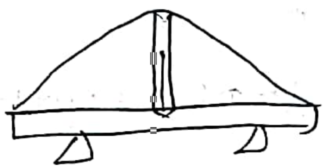


(ii) Dynamic experiment

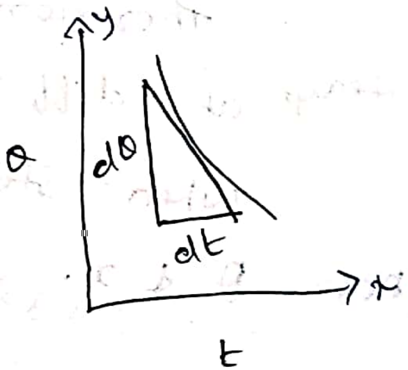
→ original rod is heated

→ suspended in air → cool.

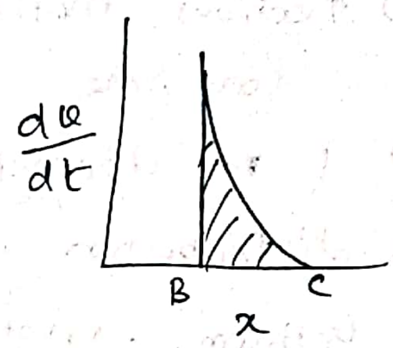
→ temp is noted at regular interval of time



$\frac{d\theta}{dt}$ → from graph



3rd graph



The area bounded by the curve is

$$\int_B^C \frac{d\theta}{dt}\,dx. \Rightarrow \text{Area of shaded Portion}$$

We K.T

$$K = \frac{\rho s \int_B^C \frac{d\theta}{dt}\,dx}{\left(\frac{d\theta}{dx}\right)_B}$$

$$= \frac{\rho s \times (\text{Area of shaded portion})}{\tan\alpha.}$$

Merits

→ earliest method.

→ validates fundamental eqn.

Demerits

→ tedious method - take long time

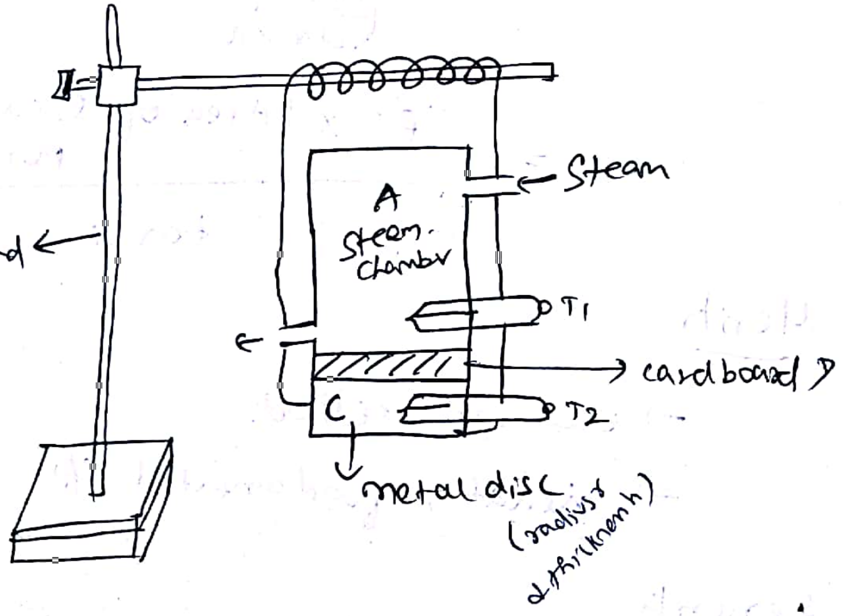
→ s ~~x~~ doesnot remains constant

→ distribution of heat is not uniform. → not accurate.

Lee's disc Method

glass, ebonite, card board

Description



radius of disc - r

thickness of disc - h

Thickness of card board - d

Mass of slab - M

Steam chamber temp - θ_1

slab " - θ_2

Thermal conductivity - k

Rate of cooling - R

Specific heat capacity - s

Area - πr^2

Amount of heat conducted per sec

$$Q = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$= \frac{k \pi r^2 (\theta_1 - \theta_2)}{d} \quad -(1)$$

Amount of heat lost = M × s.h.c × rate of cooling

$$= MSR \quad -(2)$$

At steady state

Heat conducted = Heat lost

$$\frac{K \pi r^2 (\theta_1 - \theta_2)}{d} = MSR$$

$$K = \frac{MSRd}{\pi r^2 (\theta_1 - \theta_2)} \quad \text{—(3)}$$

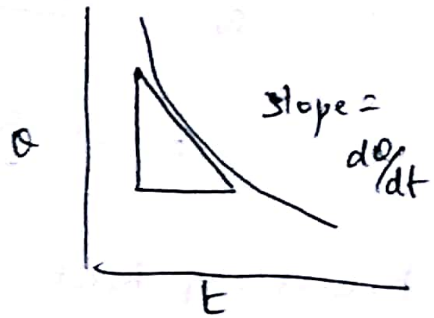
Determination of Rate of cooling.

Card board removed

$5^{\circ}C$.

slab alone to cool

Temp is noted at regular interval of time.



1^{st} part of experiment :

Radiation emitted from bottom surface area & curved surface area

Total area = $\pi r^2 + 2\pi r h$

$= \pi r (r + 2h)$

2^{nd} part

radiation → bottom, top & curved

Total area = $\pi r^2 + \pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r h$

$= 2\pi r (r + h)$

Rate of cooling is ∝ to area.

$$\frac{R}{\frac{d\theta}{dt}} = \frac{\pi r (r + 2h)}{2\pi r (r + h)}$$

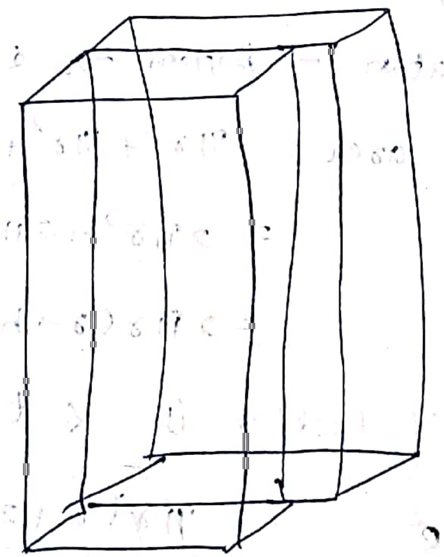
$$R = \frac{d\theta}{dt} \left(\frac{r + 2h}{2r + 2h} \right)$$

$$K = \frac{MS\left(\frac{dQ}{dt}\right) d}{\pi r^2 (\theta_1 - \theta_2)} \frac{r+2h}{2r+2h}$$

$Wm^{-1}K^{-1}$

X ——— X

Problems



UNIT 3 - Thermal physics

Thermal Insulation

* To resist the flow of heat to & from a body

Principle

* Thermal resistance of insulating material is ∝ to thickness.
* Air gap is important insulating agent
* Thermal resistance of building depends on orientation.
* Heat is transferred by conduction, Convection or radiation.

Reducing heat transfer by

(i) Conduction

→ rate at which heat flow is depends on thermal conductivity.

→ when K ↑, Poorer is the thermal insulation.

$$K = \frac{MS\left(\frac{dQ}{dt}\right) d}{\ldots \quad r+2h}$$

(ii) Convection.

→ If air space within the walls (material containing pores: sponges, wood, porous material) of a house is filled with porous material, air circulation will be impeded (delayed), hence heat transfer is reduced.

(iii) Radiation

* depends on temperature of the surface & kinds of surface.

Thermal insulating Materials

The materials which are used to insulate thermally.

Types:
- → organic materials ex: Silk, wool, sugarcane fiber etc
- → Inorganic materials ex: Gypsum powder, coke powder, asbestos etc.

Types of thermal Insulation

- → House T.I
- → Industrial TI
- → Building TI

(i) House Thermal Insulation

* Warm house during winter → insulation in walls, ceilings & floors reduces the loss of heat.

* Cool house during summer → insulation reduces the entry of heat.

(ii) Industrial thermal Insulation

TI is used for

→ In Industry, enclosing heating equipments & pipes etc.

→ The insulation helps to maintain uniform temp & to conserve fuel.

(iii) Building Thermal Insulation

exposed doors & windows.

→ insulating glass or double glass with air space.

→ weathershed curtains.

→ flat roofs → kept cool by water either stored or sprayed at regular intervals., also putting a layer of 25mm thickness of coconut pith.

Thermal Insulation of exposed walls

→ suitable thickness of wall.

→ hollow wall - adopted

→ for partition, air space may be created.

Thermal Insulation of exposed roofs

It may achieved either inside surface or outside surface

→ false ceilings with air gap.

→ light insulating materials may be pasted by suitable adhesive - inside surface